B A Short Introduction to Decibels

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Prerequisite knowledge required: None

B.1 Introduction

Decibels are a way of writing very large (or very small) power ratios without needing to use exponential notation, metric multipliers[1] (e.g. 'n' for 10^{-9} or 'M' for 10^{6}) or unwieldy quantities of zeros (e.g. writing 12n as 0.000000012). This is achieved by using the logarithm of the ratio, rather than as the ratio itself.

To start with a definition: the decibel is ten times the logarithm (to the base 10) of a power ratio[2]. Remember that it's a *power ratio*, always a *ratio* of *powers*, and you won't go far wrong. (Most of the difficulty in using decibels comes from trying to apply the concept to parameters which are not the ratio of powers.)

For example: suppose you had an amplifier with an input signal of 2 mW and an output power of 20 W. The *power gain* (often just called the *gain*[3], or sometimes the *linear gain* to emphasise that the figure is not in decibels) is the output power divided by the input power:

$$\text{power gain} = \frac{\text{output power}}{\text{input power}} = \frac{20\ \text{W}}{0.002\ \text{W}} = 10000 \tag{B.1}$$

Since this is a ratio of two powers, we can also express it in dB:

$$\text{power gain in dB} = 10 \times \log_{10}\left(\frac{20}{0.002}\right) = 10 \times \log_{10}(10000) = 40\ \text{dB} \tag{B.2}$$

That's it. It's not that difficult. However, there is a huge amount of confusion surrounding the use of the dB, most of which arises due to some short-cuts made by engineers in a particular field of electronics who do not always state all of their assumptions and leave some things as ambiguous for

[1] What's wrong with metric multipliers? Nothing really; it's just that if you want to work out how much bigger a signal gets when passing through an amplifier, or how much smaller when passing through a long cable, you have to do a multiplication or a division. It's much easier to do additions and subtractions (at least it was in the time long before pocket calculators when decibels were invented), and since decibels are logarithms, adding them is equivalent to multiplying the actual numbers, and subtracting them is equivalent to dividing the actual numbers.

[2] It derives from a unit called a 'bel' (or B for short), which was just the logarithm (to the base ten) of the power ratio. However, this was found to be a rather large unit, so the deci-bel, which is one-tenth of a bel, was used instead. This is why 'dB' is always written with a lower-case 'd' (for deci) and an upper-case 'B' (short for bel). It's similar to how 'kHz' (short for kilohertz) has a capital 'H', since the short form of 'hertz' is 'Hz'.

[3] Although be careful here: there is another quantity (the voltage gain, which is the ratio of the output voltage to the input voltage) which is also often called just "the gain". The power gain and the voltage gain are not the same, and to avoid ambiguity, it's a good idea to always refer to the "power gain" or the "voltage gain", and never just "the gain".

anyone who doesn't know what assumptions they are making, but more about that later. First, I'll do some simple examples.

It's very useful to know the corresponding values in dB and linear power ratios for some common cases: for example a linear power ratio of ten is 10 dB, and a linear power ratio of 2 is 3 dB[4].

$$10 \times \log_{10}(10) = 10 \tag{B.3}$$

$$10 \times \log_{10}(2) = 3.01 \approx 3 \tag{B.4}$$

Engineers often talk about a signal being '3 dB down' when they mean it's got about half the power, and it's useful to know this without having to reach for your calculator. Similarly, '6 dB up' means about four times the power, since:

$$10 \times \log_{10}(4) = 6.02 \approx 6 \tag{B.5}$$

B.2 Using decibels

Since dBs are logarithmic units, they behave just like logarithms: if you want to multiply two linear power ratios, you just add the dB together; if you want to divide two linear power ratios, you can just subtract the dB.

For example: 10 dB minus 3 dB is 7 dB, and a subtraction in dBs is equivalent to a division in linear power ratios, and 10 dB is a linear factor of 10 and 3 dB is a linear factor of approximately 2, so 7 dB must be a linear factor of approximately 10 / 2 = 5. Which it is (more accurately 7 dB corresponds to a linear factor of 5.012, but again the difference is rarely important).

A few common examples worth knowing:

Linear Power Ratio	*Power Ratio in dB*
1	0 dB
0.1	-10 dB
0.01	-20 dB
10	10 dB
100	20 dB
1000	30 dB
2	3 dB (approx.)
4	6 dB (approx.)
0.5	-3 dB (approx.)

B.3 A word of warning

In some textbooks (including this one), you will see the gain in dB being calculated using a formula like this:

[4] Well, not exactly three decibels (it's more accurately expressed as 3.0103 dB) but three decibels is a good enough approximation for just about all engineering purposes.

$$\text{gain (dB)} = 20 \times \log_{10}\left(\frac{V_{out}}{V_{in}}\right) \tag{B.6}$$

Be careful if you see this. It is common practice, but it can be misleading, as it does not always give the correct answer. The problem is that V_{out} and V_{in} are not powers, but voltages. This formula relies on the (often un-stated) assumption that the impedance in the circuit at the input and the output is the same (or at least undefined).

Remember, a dB is measure of a power ratio. If all you've got is the voltage, then you need to calculate the power using:

$$\text{power in} = \frac{(V_{in})^2}{Z_{in}} \qquad \text{power out} = \frac{(V_{out})^2}{Z_{out}} \tag{B.7}$$

In the most general case, when the input voltage has an impedance Z_{in} and the output has an impedance Z_{out}, then the power gain can be expressed as:

$$\text{power gain} = \frac{\text{power out}}{\text{power in}} = \frac{(V_{out})^2}{Z_{out}} \div \frac{(V_{in})^2}{Z_{in}} = \frac{(V_{out})^2}{(V_{in})^2}\frac{Z_{in}}{Z_{out}} \tag{B.8}$$

Now if, and only if, the output impedance is equal to the input impedance, then we can write:

$$\text{power gain in dB} = 10 \times \log_{10}\frac{(V_{out})^2}{(V_{in})^2} = 10 \times \log_{10}\left(\frac{V_{out}}{V_{in}}\right)^2 = 20 \times \log_{10}\left(\frac{V_{out}}{V_{in}}\right) \tag{B.9}$$

But, and it's worth emphasising this point yet again, if the input impedance is not equal to the output impedance, then the simple formula gain (dB) = $20\log_{10}(V_{out}/V_{in})$ doesn't work. In the general case, we'd have to write:

$$\text{power gain dB} = 20 \times \log_{10}\left(\frac{V_{out}}{V_{in}}\right) - 10 \times \log_{10}\left(\frac{Z_{out}}{Z_{in}}\right) \tag{B.10}$$

In many cases in simple circuit analysis, the impedance at a certain point in the circuit is either not given, does not really exist (for example if the amplitude of a signal is stored as a digital number) or is not easily calculable. In these cases, the impedance is often normalised to be one, and the $20\log_{10}(V_{out}/V_{in})$ formula is used. It's common practice, but don't forget about the exceptions.

Before moving on, I should also mention another point of confusion about the formula for gain in terms of the voltage ratio. Some amplifiers invert their inputs, so for example an input of 1 V produces an output of -10 V. However it's not possible to take the logarithms of negative numbers (at least and get real answers), so a simple application of the formula:

$$\text{power gain in dB} = 20 \times \log_{10}\left(\frac{V_{out}}{V_{in}}\right) \tag{B.11}$$

doesn't work. In these cases it's important to remember where this formula came from:

$$\text{power gain in dB} = 10 \times \log_{10}\left(\frac{V_{out}^{2}}{V_{in}^{2}}\right) \tag{B.12}$$

and note that it's ratio of the square of the voltages that's important, and squares are always positive. It would be clearer to write the formula for dB in terms of voltage gain as:

$$\text{power gain in dB} = 20 \times \log_{10}\left(\frac{|V_{out}|}{|V_{in}|}\right) \tag{B.13}$$

since it's only the magnitude of the voltage that is important when calculating gains in dB.

B.4 Some derived units: dBmW, dBμW, dBW, etc

A dB is a unit of power ratio. If you want to express a real power in terms of dB, then you need some reference power to use, so that the power can be expressed as a ratio with the reference power. Adding a letter (or two) to the dB unit specifies what reference power is being used.

For example, a real power of 30 W could be expressed as a power that is 30 times greater than a power of one watt. In these cases, where a reference unit of one watt is assumed, the result of the ratio is expressed in terms of the units of dBW. So, to express a power in dBW:

$$\text{power dBW} = 10 \times \log_{10}\left(\frac{P(W)}{1\ W}\right) = 10 \times \log_{10} P(W) \tag{B.14}$$

where *P*(W) is the power of the signal expressed in watts.

To do a numeric example, 100 Watts is 20 dBW, since $10\log_{10}(100) = 20$, and one milliwatt (1 mW) is –30 dBW, since $10\log_{10}(0.001) = -30$. To calculate the power in dBW, just take 10 times the logarithm (to the base 10) of the ratio of the power to one Watt.

Some more examples: 2 W is 3 dBW; 10 W is 10 dBW; and 500 mW is –3 dBW.

The reference power doesn't have to be one watt (sometimes one watt is a rather large power to use, and smaller units of power are more convenient). For example, if the reference power used is one microwatt, then the units become dBμW, and a power of 1 mW could be written as 30 dBμW, since:

$$\text{power dB}\mu\text{W} = 10 \times \log_{10}\left(\frac{P}{10^{-6}}\right) = 10 \times \log_{10}\left(\frac{10^{-3}}{10^{-6}}\right) = 30\ \text{dB}\mu\text{W} \tag{B.15}$$

Similarly, using a reference power of 1 mW leads to the unit dBmW, and hence:

$$1\ \text{mW} = -30\,\text{dBW} = 0\,\text{dBmW} = 30\ \text{dB}\mu\text{W} \tag{B.16}$$

B.4.1 The dBV and dBA

dBW[5] is a unit of power, with the reference power being one watt. Powers in watts can be converted into powers in dBW using the standard formula:

$$\text{power in dBW} = 10 \times \log_{10}\left(\frac{P(\text{in Watts})}{\text{Reference Power}}\right) = 10 \times \log_{10}\left(\frac{P(W)}{1\ W}\right) = 10 \times \log_{10} P(W) \qquad \text{(B.17)}$$

In a similar way, dBV[6] is a way of expressing voltages in decibel terms, using a reference voltage of one volt. Using formula (B.6) then gives:

$$\text{voltage in dBV} = 20 \times \log_{10}\left(\frac{V(\text{in Volts})}{\text{Reference Voltage}}\right) = 20 \times \log_{10}\left(\frac{V(V)}{1\ V}\right) = 20 \times \log_{10} V(V) \qquad \text{(B.18)}$$

While I'm at it, there's a similar quantity dBA[7] which allows currents to be expressed in decibel terms as well. This uses a variant of the previous equation, except using a reference current of one amp:

$$\text{current in dBA} = 20 \times \log_{10}\left(\frac{I(\text{in Amps})}{\text{Reference Voltage}}\right) = 20 \times \log_{10}\left(\frac{I(A)}{1\ A}\right) = 20 \times \log_{10} I(A) \qquad \text{(B.19)}$$

B.4.2 The dBm, dBu and dBμ

At first sight these common units don't make any sense. A dB is a power ratio. A 'μ' isn't a unit of power or voltage or current, it just means a factor of 10^{-6}, and 'm' means 10^{-3}, and what is 'u' about? While dBm is usually shorthand for dBmW, if you thought that dBμ was just shorthand for dBμW, then in most cases you'd be wrong. It's confusing, this bit.

There are several different uses of the terms dBu or dBμ. Since occasionally the μ is written as a u (due to difficulties in typing Greek letters), often the only way you can tell what is meant is by context. For example:

(a) Audio Engineering: 'dBu' means use as a reference power 0.775 V (rms) in an impedance of 600 ohms (a standard impedance used for audio work). So in this case, the reference power is one milliwatt (since $V^2 / R = 0.775^2 / 600 = 10^{-3}$ W = 1 mW).

However, since the impedance often isn't 600 ohms, what this actually means in practice is that a voltage of rms amplitude V would be expressed as $20 \log_{10}(|V| / 0.775)$ dBu no matter what the impedance in the circuit happens to be.

(The 'u' is short for 'unloaded' in this case; it has nothing to do with an inability to type 'μ'.)

(b) Radio propagation/EMC: Here, 'dBμ' is most often used as a shorthand for dBμV/m. It's impossible to define a reference power in this case although you can define a reference

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[5]pronounced "dee-bee-watts" or "dee-bee-double-yew".

[6] pronounced "dee-bee-vee" or "dee-bee-volts"

[7] pronounced "dee-bee-a" or "dee-bee-amps")

power density (in Watts per square metre). In the case of free space, it corresponds to a power density of 2.65 10^{-15} W/m^2.

(c) Other fields: 'dBμ' is a shorthand for dBμW. In this case the reference power is one microwatt.

It is confusing. But if you remember that a dB is always calculated using a *power* ratio, you should be fine; you just have to know / remember / work out what the reference *power* is in each case (or if there isn't a reference power, use 20 $\log_{10}$(V / ref) with the appropriate reference voltage).

B.5 Using deciBels

If the decibel is so confusing, why does anyone bother using them? The answer is that they make calculations much easier by allowing multiplications and divisions to be replaced by additions and subtractions. Before the days of pocket calculators that made a lot of difference, and it's still easier to add and subtract in your head than to multiply and divide.

It's also convenient to be able to express a very wide range of possible values without having to use exponents, or a large number of zeros before or after the decimal points. (It's very easy to miscount the number of zeros in a large number, and that could be disastrous.)

Many formulas can be expressed conveniently in terms of dB by taking the logarithm of the formula and multiplying by ten. For example, consider the following formula for the received power into a matched receiver for a radio link in free space:

$$P_r = \frac{P_t G_t G_r}{(4\pi d / \lambda)^2} \tag{B.20}$$

where P_r is the received power, P_t is the transmitted power, G_t and G_r are the gains of the transmit and receive antennas, d is the distance between the antennas and λ is the wavelength. Take ten times the logarithm (to the base ten) of this equation, and we get:

$$10\log_{10}(P_r) = 10\times\log_{10}(P_t) + 10\times\log_{10}(G_t) + 10\times\log_{10}(G_r) - 20\times\log_{10}(4\pi d/\lambda) \tag{B.21}$$

If the transmit power P_t and receive power P_r are given in Watts, then we can write this as:

$$P_r\ (\text{dBW}) = P_t\ (\text{dBW}) + G_t\ (\text{dB}) + G_r\ (\text{dB}) - 20\times\log_{10}(4\pi d/\lambda) \tag{B.22}$$

Specifying the gains of the antennas in decibels, and the transmitted and received powers in dBW saves a lot of multiplication and division, replacing these operations with additions and subtractions, and speeding up the calculations significantly[8].

Note that the power transmitted and power received are expressed in terms of dBW, but the gains of the antennas are expressed in terms of dB. Since antenna gains are defined in terms of the ratio of two powers (the power in a given direction divided by the power that would have been

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[8] Admittedly you do have to look up the logarithm of the term $4\pi d/\lambda$, but before pocket calculators multiplications and divisions were often done by taking logarithms, adding and subtracting, and then taking the antilogarithm of the answer, so this would have to be done anyway.

transmitted in this direction by an omnidirectional antenna), this is fine: we don't need a reference power for a simple power gain.

Note that we could equally well have written:

$$P_r\left(\text{dB}\mu\text{W}\right) = 60 + P_t\left(\text{dBW}\right) + G_t\left(\text{dB}\right) + G_r\left(\text{dB}\right) - 20\times\log_{10}\left(4\pi d/\lambda\right) \quad \text{(B.23)}$$

since 60 dBμW = 0 dBW, and 60 + P_t dBW = P_t dBμW. Similarly:

$$P_r\left(\text{dB}\mu\text{W}\right) = 30 + P_t\left(\text{dBmW}\right) + G_t\left(\text{dB}\right) + G_r\left(\text{dB}\right) - 20\times\log_{10}\left(4\pi d/\lambda\right) \quad \text{(B.24)}$$

since 30 + P_t dBW = P_t dBmW.

B.5.1 Adding dB

Adding two quantities in dB is done all the time, this just results in another quantity also in dB. Adding one quantity in dB to one quantity in dBu (or dBm) is also very common, and results in another quantity also in dBu (or dBm). Adding two quantities in dBu or dBm is extremely rare, and if you find yourself doing this, you are almost certainly doing something wrong.

Why? Remember that a dB is a power ratio: it doesn't have any dimensions. It's used to express the gain of a system: the ratio of the output power to the input power. And since decibels are logarithmic quantities, adding two decibels is effectively multiplying the two power gains together:

$$10\times\log_{10}\left(G_1\times G_2\right) = 10\times\log_{10}\left(G_1\right) + 10\times\log_{10}\left(G_2\right) \quad \text{(B.25)}$$

For example: you have two amplifiers, one with a gain of 10 (10 dB) and another with a gain of two (3 dB). You put them in series, what is the total gain of the combined two-stage amplifier?

$$\begin{aligned} \text{Linear Power Gain} &= 10\times 2 = 20 \\ \text{Power Gain (dB)} &= 10\times\log_{10}\left(10\times 2\right) \\ &= 10\times\log_{10}\left(10\right) + 10\times\log_{10}\left(2\right) \\ &= 10\text{ dB} + 3\text{ dB} \\ &= 13\text{ dB} \end{aligned} \quad \text{(B.26)}$$

All you have to do is add the gains in dB of the two stages to get the total gain. The same is true for any number of stages of amplification: just add all the gains of the individual stages in dB.

B.5.2 Adding dB and dBm

What about adding quantities in dBm? Adding one quantity in dBm to another in dB is equivalent to multiplying a power by a gain: remember dB are logarithmic, so adding them together is equivalent to multiplying the linear quantities, and a dBm is a power, whereas a dB is just a gain. Multiplying a power by a gain is a very common thing to want to do; this is how to find out how much power is coming out of a system with a particular gain.

For example, consider you have a 10 W signal, and it's being input into a system with a power gain of 0.01 (in other words only 1% of the power comes out). How much power comes out? In linear units, all you do is multiply the input power with the power gain:

$$\text{Output Power} = 10 \times 0.01 = 0.1 \text{ W} \tag{B.27}$$

Working in dB, this becomes:

$$\begin{aligned} 10\log_{10}(\text{Output Power}) &= 10 \times \log_{10}(10 \times 0.01) \\ &= 10 \times \log_{10}(10) + 10 \times \log_{10}(0.01) \\ &= 10 \text{ dBW} + (-20 \text{ dB}) \\ &= -10 \text{ dBW} \end{aligned} \tag{B.28}$$

In other words, you just add the input power in dBW to the gain in dB, and you get the output power in dBW.

B.5.3 Adding dBW and dBW

What about adding two quantities both in dBW, or one quantity in dBW with another in dBm? This is much more rarely done. Anything expressed in dBW or dBm is a power, and adding them together would be the equivalent of multiplying two powers together, which would give a result with units of watts-squared. It is difficult to express that using dB. If you find yourself having to multiply two powers together, it's usually better not to use dB.

This is a useful rule to remember: if you find you are adding two quantities both of which are expressed in dBm, dBW or dBu, you are probably doing something wrong. Make sure you really want to multiply two powers.

B.5.4 Adding powers expressed in dBm

Suppose you have two powers both expressed in dBm, and you want to know what the total power is. How do you do that? Answer: you have to convert the powers to mW, then add them up, then convert the answer back into dBm. It's long-winded, but there's no other way to do it.

This is a common mistake: for example what do you get if you have a 20 dBm signal, and you add 20 dBm of noise power? What is the total power? The answer is 23 dBm, since:

$$\begin{aligned} \text{Total Power (mW)} &= 10^{(20/10)} + 10^{(20/10)} \\ &= 100 + 100 \\ &= 200 \text{ mW} \end{aligned} \tag{B.29}$$

and

$$\begin{aligned} \text{Total Power (dBm)} &= 10 \times \log_{10}(200) \\ &= 23 \text{ dBm} \end{aligned} \tag{B.30}$$

(The answer is definitely not 40 dBm. That would be the equivalent of saying that a 100 mW signal plus another 100 mW signal had a power of 10 watts. You can get some very silly answers if you get this wrong.)

B.5.5 Multiplying and dividing quantities expressed in dB and dBm

In general, don't do it! You're almost certainly doing something wrong[9]. Since decibels are logarithmic quantities, multiplying them together is the equivalent to raising to the power of another, or taking the n^{th} root of a power. I can't think of many situations in which anyone would want to do that.

If in any doubt, take the quantities out of dB, then do the calculations, then put the answer back into dB (or dBm, dBW or whatever). It takes longer, but it's safer (and sometimes there is no alternative).

B.6 Some further examples

1) An amplifier has a power input of 2 Watts, and an output power of 8 Watts. What is the gain in dB?

$$10 \times \log_{10}\left(\frac{8}{2}\right) = 10 \times \log_{10}(4) = 6.02 \text{ dB}$$

In practice, it's usually just quoted as 6 dB. This is a very useful thing to know – a factor of two in power is 3 dB, a factor of four in power is 6 dB, a factor of eight in power is 9 dB, etc. This is accurate enough for most purposes.

2) An amplifier has a voltage input of 2 Volts, and an output voltage of 8 Volts. What is the gain in dB?

Strictly speaking, a correct answer would be "it's impossible to tell". You are not told what the input and output impedances are, so you can't tell what the input power or output power are.

However, if you are told (or can assume) that the output impedance is equal to the input impedance, or you are not considering the impedances, then the gain could be calculated as:

$$10 \times \log\left(\frac{8^2}{2^2}\right) = 10 \times \log\left(\frac{8}{2}\right)^2 = 20 \times \log(4) = 12 \text{ dB} \qquad \text{(B.31)}$$

3) An amplifier has a voltage input of 3 Volts with an 50-ohm impedance, and supplies an output voltage of 4 Volts into a 100-ohm impedance. What is the gain in dB?

$$10 \times \log_{10}\left(\frac{P_{out}}{P_{in}}\right) = 10 \times \log_{10}\left(\frac{V_{out}^2}{R_{out}}\frac{R_{in}}{V_{in}^2}\right) = 10 \times \log_{10}\left(\frac{4^2}{100}\frac{50}{3^2}\right) = \text{-0.51 dB}$$

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[9] There is one common exception to this rule: if you're using the log-normal distribution to represent a distribution of channel gains or signal powers (this is very common with mobile radio channels), and using the Q or erf functions to calculate the probability of a fade. In these cases, the standard deviation is expressed in dB, and to work out the argument of the Q-function requires a division by the standard deviation in dB. However, that's about the only case I can think of where two quantities both in dB are multiplied or divided.

(If that makes no sense to you yet, don't worry. Just don't multiply or divide any quantity expressed in dB or dBm by anything.)

Note that even though the voltage at the output is greater than the voltage at the input, the power supplied by the output is less than the power received from the input, so the gain in dB is negative.

4) Convert the formula $E^2 = P / 120\pi$ into decibel form, where P is the power in Watts and E is the electric field strength in V/m.

Take 10 times the $\log_{10}$ of both sides, and we get:

$$10 \times \log_{10}\left(E^2\right) = 10 \times \log_{10}(P) - 10 \times \log_{10}(120\pi)$$

$$20 \times \log_{10}(E) = P\ (\text{dBW}) - 25.76$$

5) Express a power of 70 dBμW in dBW, dBmW, and dBμV assuming an impedance of 100 ohms.

$$70\ (\text{dB}\mu\text{W}) = 10 \times \log_{10}\left(\frac{P(\text{W})}{10^{-6}(\text{W})}\right)$$

$$P = 10^7\ \mu\text{W} = 10\ \text{W}$$

Therefore,

$$P\ (\text{dBW}) = 10 \times \log_{10}\left(\frac{P(\text{W})}{1(\text{W})}\right) = 10 \times \log_{10}(10) = 10\ \text{dBW}$$

$$P\ (\text{dBmW}) = 10 \times \log_{10}\left(\frac{10(\text{W})}{10^{-3}(\text{W})}\right) = 10 \times \log_{10}\left(10^4\right) = 40\ \text{dBmW}$$

$$P\ (\text{dB}\mu\text{V in 100 ohms}) = 10 \times \log_{10}\left(\frac{10(\text{W})}{\left(10^{-6}\right)^2 / 100}\right) = 10 \times \log_{10}\left(10^{15}\right) = 150\ \ \text{dB}\mu\text{V}$$