28 A Short Introduction to Non-Linear Operational Amplifier Circuits

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Prerequisite knowledge required: Op-Amps, Ohm and Kirchhoff's Laws, Circuit Loading

28.1 Introduction

Most op-amps are employed in circuits that use negative feedback to produce highly linear amplifiers. However, there are also many useful applications of op-amps in non-linear circuits, where a doubling of the input does not result in a doubling of the output. This note introduces a few of the more common and interesting of these circuits, including:

- A circuit that compares two voltages (the comparator)
- An improvement on the comparator which is less sensitive to noise (the Schmitt trigger)
- A circuit that outputs the modulus of the input (the precision rectifier)
- Circuits that can multiply or divide voltage, take logs and exponentials

although I should point out that there are far more clever and inventive uses for op-amps than I have room to write about here.

28.2 The comparator

The role of a comparator is to compare two voltage levels and produce an output signal dependent only on which input is at the higher voltage. It isn't linear: the output of an ideal comparator is either in one state or the other, but never anywhere else in-between.

You can make a basic comparator using an op-amp, just by leaving off the feedback resistor:

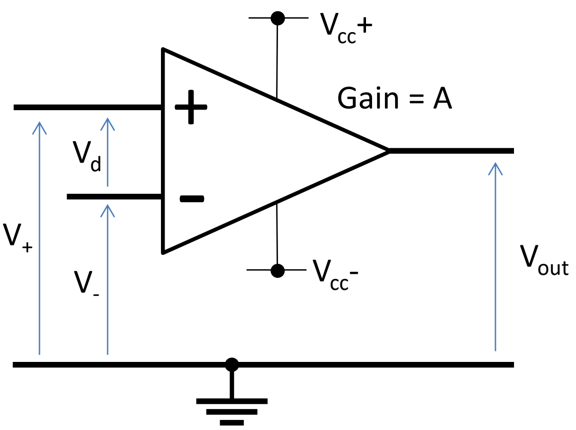

Figure 28.1 Op-Amp as a comparator

In theory, the output of such an op-amp will try to reach:

$$V_{out} = A\,V_d = A\left(V_+ - V_-\right) \tag{28.1}$$

and since A is typically a very large gain, providing no feedback at all results in the op-amp attempting to produce a very high output voltage when $V_+ > V_-$ and a very low output voltage when $V_+ < V_-$. However, since all op-amps have a limited output range (limited by their power supply), the effect in practice is to send the op-amp into positive saturation (where the output voltage is close to the positive power supply) when $V_+ > V_-$, and into negative saturation (where the output voltage is close to the negative power supply) when $V_+ < V_-$.

It's only when the difference between the two inputs is less[1] than V_{CC} / A (which is around 100 µV for typical values of V_{CC} and A) that the output of this sort of comparator is not at one of the extreme values of voltage and the circuit does not behave like an ideal comparator.

28.2.1 Practical comparators and open-collector outputs

In real-life, this idea of using an op-amp as a comparator is rare: there are specialist comparator chips available that are optimised for this function[2]. Many of these don't provide a voltage at their output, instead they provide what is known as an *open-collector output*. You can think of this as effectively like a switch: the output is either connected to ground or left floating:

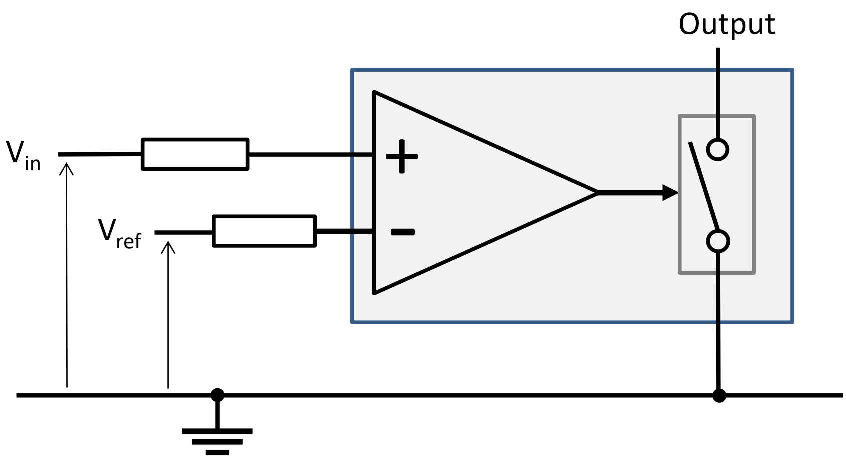

Figure 28.2 Open-collector comparator operation

Note that as well as the positive and negative power supplies (not shown in the figure above) a comparator chip like this also needs a connection to ground (unlike most op-amps).

To use these comparators in a circuit, it's important to provide a pull-up resistor, so that the output from the comparator is dragged high by the resistor when the switch[3] is turned off.

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[1] I'm assuming that the op-amp is ideal apart from the finite gain here. In particular, I'm neglecting the effect of the input offset voltage (see the chapter on non-ideal op-amps for more details).

[2] They have particularly fast switching times, low offset voltages, and don't have to worry about providing linear gain.

[3] It's not a real switch of course, it's either the collector terminal of a bipolar transistor, or the drain terminal of a field-effect transistor. However, the operation of transistors is beyond the scope of this module, and for now you can think of this as a switch.

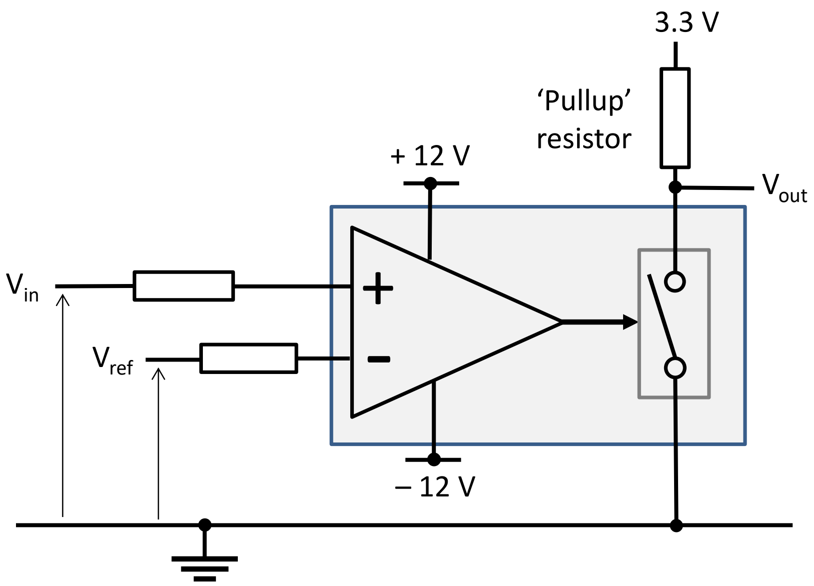

Figure 28.3 Open-collector operation showing the pull-up resistor

This type of circuit output might look unhelpful at first sight, but it has the big advantage that the voltage output of the comparator doesn't have to be close to the negative or positive rails of the comparator. For example, a circuit such as that shown above can accept input voltages anywhere between –12V and +12V (the negative and positive supplies), and provides an output which is either close to ground, or close to +3.3V. This makes it much easier to interface to digital logic which uses 0V and 3.3V to represent a '0' and a '1'.

28.3 The Schmitt trigger

The Schmitt trigger is an extension of the idea of a comparator, but instead of using the negative feedback used in linear op-amp amplifiers, positive feedback is used. This means that if the output of the op-amp starts to increase, the positive feedback will act to accelerate this movement upwards, and the op-amp will move into positive saturation more quickly (and the equivalent thing happens when moving into negative saturation).

However this additional speed of switching is not the only (nor the most interesting) effect of introducing positive feedback into a comparator circuit. Consider the circuit shown below:

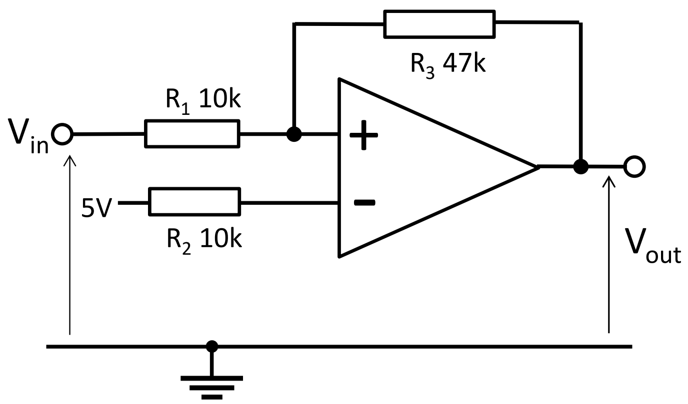

Figure 28.4 A Schmitt trigger circuit

This circuit was designed to output a large positive voltage when the input V_{in} is greater than 5V, and a large negative voltage when the input is less than 5V.

However, consider what happens when the input V_{in} is held equal to 5V. The output could be in either positive or negative saturation, but nowhere in-between. This circuit is always in saturation, even if the two inputs are held at exactly the same voltage. This can be very useful if the comparator is trying to produce an input for a digital circuit, where you don't want the voltage to be anything other than a low voltage representing a logic '0' or a high voltage representing a logic '1'.

First, consider if the output is positive. The voltage at the non-inverting input is the centre-point of a potential divider with V_{out} at one end and V_{in} at the other (see Figure 28.5):

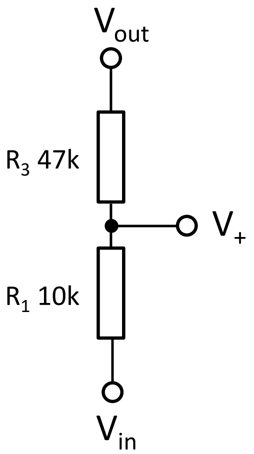

Figure 28.5 Positive feedback potential divider

This suggests that when V_{out} is at the maximum possible value (perhaps around 12 V), the voltage at the non-inverting input of the op-amp would be:

$$\begin{aligned} V_+ &= V_{in} + (V_{out} - V_{in}) \frac{R_1}{R_1 + R_3} \\ &= 5 + (12 - 5) \frac{10}{47 + 10} = 6.2\,\text{V} \end{aligned} \tag{28.2}$$

which is above 5V, so the circuit is stable with this positive voltage at the output.

On the other hand, if V_{out} is at the minimum possible voltage (perhaps around minus 12 V), and the input voltage is still at 5V, the non-inverting input would be at:

$$V_+ = V_{in} + (V_{out} - V_{in}) \frac{R_1}{R_1 + R_3} = 5 + (-12 - 5) \frac{10}{47 + 10} = 2.0\,\text{V} \tag{28.3}$$

which is below 5V, maintaining the output at this negative voltage.

In other words, if the output is positive, then the output will remain positive, since the voltage on the non-inverting input is above the voltage on the inverting input (held at 5 V). On the other hand, if the output is negative, then the output will remain negative, since the voltage on the non-inverting input is now below the voltage on the inverting input.

What happens if the output voltage is positive, but the input then reduces to, say, 4 V?

$$V_+ = V_{in} + (V_{out} - V_{in}) \frac{R_1}{R_1 + R_3} = 4 + (12 - 4) \frac{10}{47 + 10} = 5.4\,\text{V} \tag{28.4}$$

The voltage on the non-inverting input remains above 5 V, so the output will remain positive.

How about if the input lowers still further to 3 V?

$$V_{+} = V_{in} + \left(V_{out} - V_{in}\right)\frac{R_1}{R_1 + R_3} = 3 + \left(12 - 3\right)\frac{10}{47 + 10} = 4.6\,\mathrm{V} \tag{28.5}$$

Ah! Now the voltage on the non-inverting input is below 5 V, so the output will switch over and move down to a low negative value, perhaps around -12 V. Which means the voltage on the non-inverting input is now:

$$V_{+} = V_{in} + \left(V_{out} - V_{in}\right)\frac{R_1}{R_1 + R_3} = 3 + \left(-12 - 3\right)\frac{10}{47 + 10} = 0.4\,\mathrm{V} \tag{28.6}$$

and if the input now moves back up to, say 6 V?

$$V_{+} = V_{in} + \left(V_{out} - V_{in}\right)\frac{R_1}{R_1 + R_3} = 6 + \left(-12 - 6\right)\frac{10}{47 + 10} = 2.8\,\mathrm{V} \tag{28.7}$$

which is still below 5 V, so the output will remain in the negative state.

If you plot the input and output of this circuit as the voltage at the input moves from 0 to 10 V and back again, the result would look something like this:

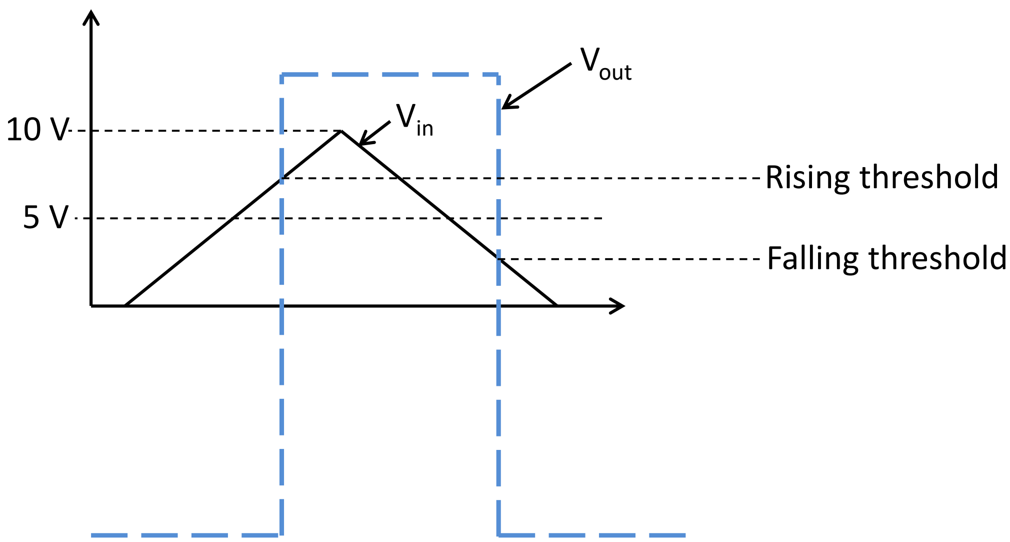

Figure 28.6 Input/output characteristic of a Schmitt trigger

The circuit behaves as if it's a comparator with two switching thresholds: one for rising voltages and one for falling voltages.

What's the point of that? Well, imagine that the input waveform is very noisy. With no positive feedback, so just a single threshold at 5V, the response of the circuit would look something like this:

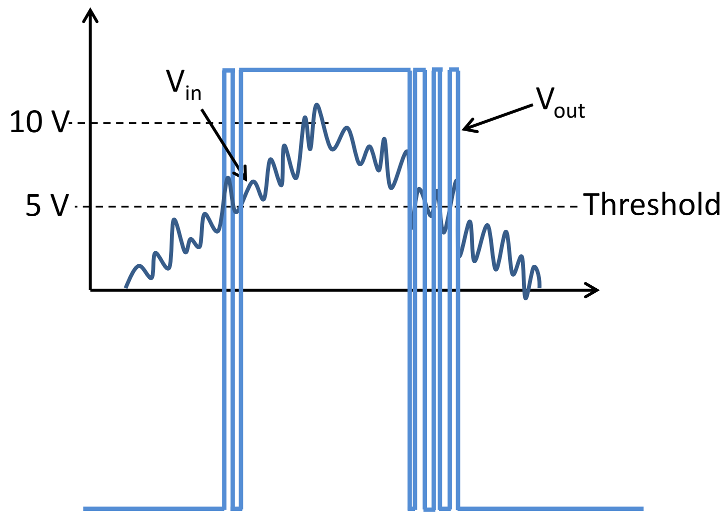

Figure 28.7 Comparator response with a noisy input

This is usually a bad thing, especially if the output from the comparator is feeding a circuit that is counting the number of rising edges. The same input waveform with some positive feedback would result in a response like that shown in Figure 28.8.

The disadvantage of using a Schmitt trigger circuit like this is the added delay: in this case the output goes high a little time after the input first rises above 5 V, but often this isn't a major problem, and the advantages of the reduction in the sensitivity to noise more than make up for this issue.

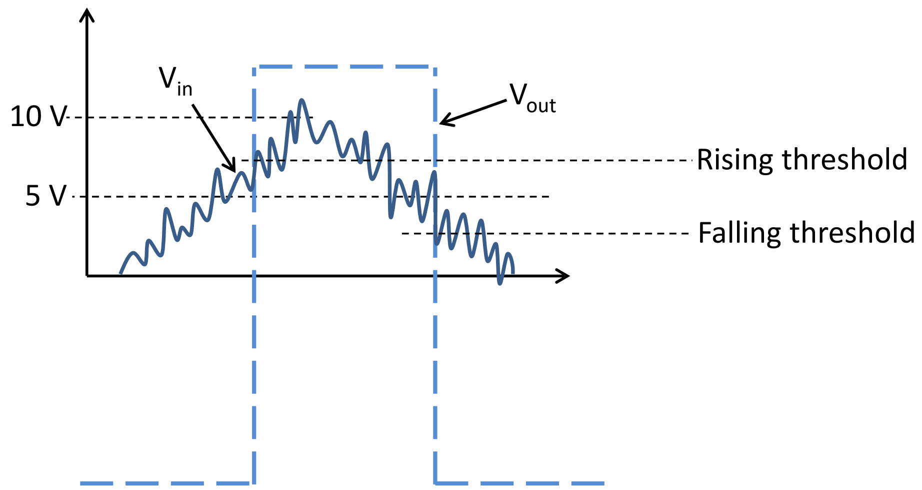

Figure 28.8 Schmitt trigger with a noisy input

28.4 Precision rectifiers

A rectifier circuit produces an output that is always positive, irrespective of the polarity of the input. Half-wave rectifiers output a copy of the input when the input is positive and zero when the input is negative; full-wave rectifiers output a copy of the input when the input is positive and minus one times the input when the input is negative (in other words they invert the input when the input is negative).

You can make a simple half-wave rectifier with a diode and a resistor, like this:

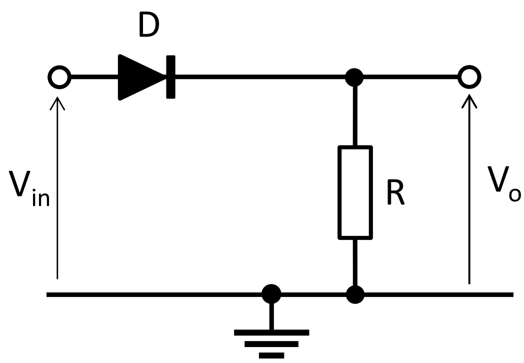

Figure 28.9 Very simple half-wave rectifier

However this isn't very accurate for several reasons:

- The output will always be lower than the input due to the voltage dropped across the diode
- The voltage dropped across the diode is a function of the size of the input voltage (the higher the input voltage, the more current flows though the diode, and hence the greater the voltage drop across the diode)
- The output voltage is a function of the input source impedance. Effectively you're building a potential divider:

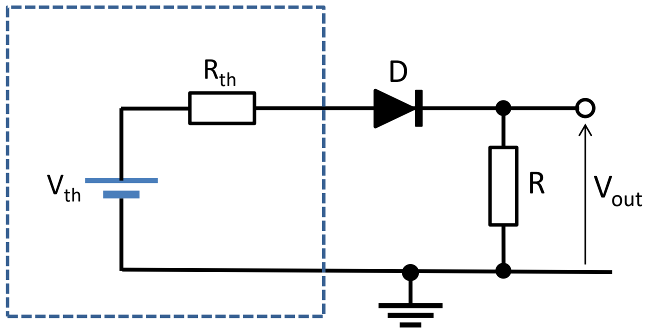

Figure 28.10 One problem with the simple half-wave rectifier

You can reduce the second problem by putting a buffer before the rectifier, effectively reducing the source impedance to such a small value that it makes no difference, like this:

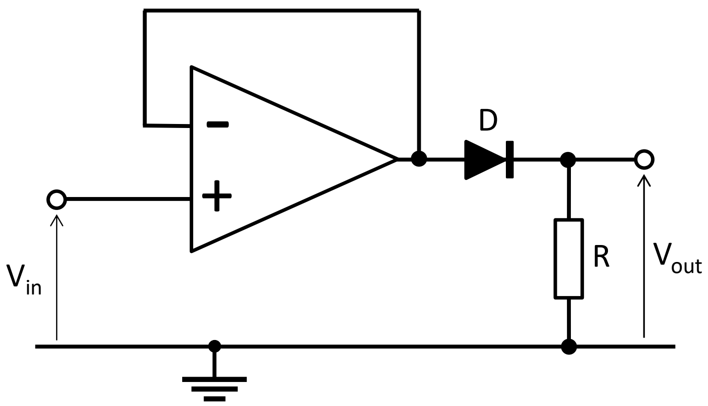

Figure 28.11 A slightly better half-wave rectifier

but the first two problems are a little harder to solve. However, consider the circuit shown below:

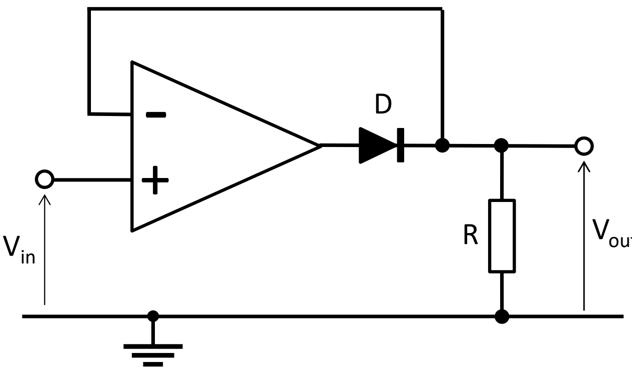

Figure 28.12 A non-inverting precision half-wave rectifier

When the input is positive (above zero volts), the output of the op-amp increases until the right-hand side (cathode) of the diode is equal to the input voltage V_{in}, so there is now no problem with the voltage dropped across the diode any more: the output of the op-amp will be maintained one diode drop above the input so that the output is equal to the input. This rather neatly solves both of the issues identified with the circuit in Figure 28.11.

However, there is a slight problem with this circuit…

28.4.1 A non-saturating precision rectifier

The problem with the precision rectifier shown in Figure 28.12 becomes apparent when you think about what happens when the input goes negative. The inverting input can't go below zero since this would require current to flow up through the resistor, and this current can't flow anywhere since no current flows into the op-amp's inverting input and the diode only allows current to flow in one direction. With no current flowing up through the resistor, the inverting input is held at ground[4].

In this case the op-amp will have a significant voltage difference between the non-inverting and inverting inputs, and this causes the output of the op-amp to go into negative saturation, and output a voltage close to the negative power supply[5].

The problem with this is that when the input returns above zero, the output from the op-amp has to move all the way from close to the negative power supply back up to a positive voltage, and this can take some time[6], resulting in a delay in the response of the circuit to an input which suddenly becomes positive. One way of avoiding this problem is to use a circuit similar to that shown below:

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[4] Again I am assuming an ideal op-amp and diode here. In practice there would be a small saturation current flowing through the reverse-biased diode, so the op-amp's output and inverting input would be very slightly below ground.

[5] As close as the op-amp can get to it, anyway; remember most op-amps can't put their output voltage all the way down to the negative power supply voltage.

[6] The voltage at the output of an op-amp has a maximum rate of change known as the slew-rate.

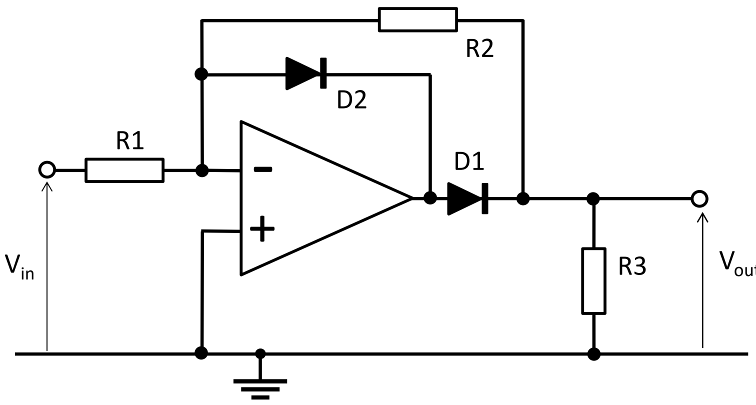

Figure 28.13 A better inverting half-wave precision rectifier

This is an inverting half-wave precision rectifier: it outputs a positive voltage when the input is negative, and outputs zero when the input is positive.

When the input is positive, most of the current flowing through R1 towards the inverting input of the op-amp flows through diode D2 to the output of the op-amp, which is held at one diode drop below ground by the negative feedback through this diode. In this case diode D1 is reverse biased, so both ends of R2 and R3 are held at ground and the op-amp never goes into saturation.

When the input is negative, D2 is reverse biased, and the circuit behaves like an inverting amplifier with a gain of – R2 / R1, giving a positive output proportional to the magnitude of the input R1. However, this circuit still only responds to half of the incoming cycle.

28.4.2 Full-wave rectifier

What if you want to produce a full-wave rectified? A circuit that really does output the modulus of the input voltage? There are a number of very clever op-amp circuits that can achieve this. For example, consider the circuit shown below:

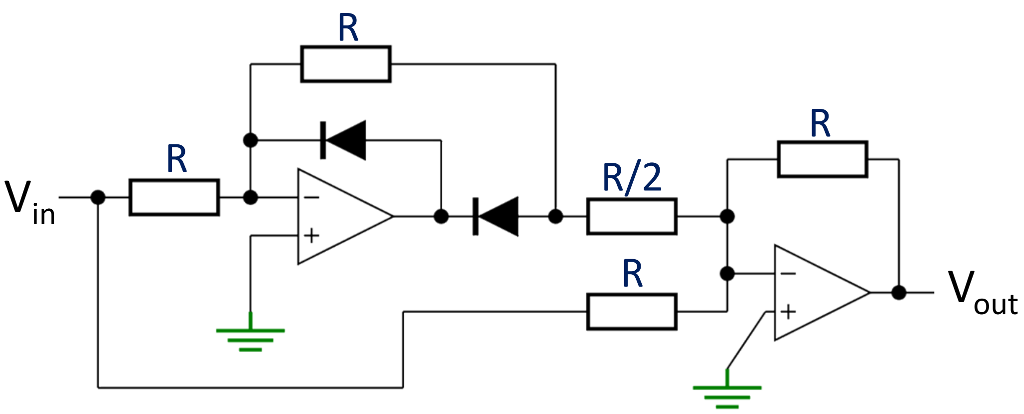

Figure 28.14 A full-wave precision rectifier

This is a full-wave rectifier, which outputs $V_{out} = V_{on}$ when $V_{in} > 0$, but $V_{out} = -V_{in}$ when $V_{in} < 0$. Perhaps the easiest way to understand how this circuit operates is to note that the second (right-hand) op-amp is an inverting summing amplifier with two inputs: one comes directly from the input (with a gain of minus one) and one from the output of the first op-amp circuit (with a gain of minus two).

The first op-amp is a half-wave rectifier which outputs minus one times the input signal when the input is positive and zero when the input is negative.

So the output when V_{in} is positive is:

$$V_{out} = -\left(V_{in} + 2 \times -V_{in}\right) = V_{in} \tag{28.8}$$

and when V_{in} is negative is:

$$V_{out} = -\left(V_{in} + 2 \times 0\right) = -V_{in} \tag{28.9}$$

which is exactly what is required for a full-wave rectifier: in both cases V_{out} is positive.

There are lots of other full-wave rectifier circuits, including some clever circuits which can accept negative-going inputs without requiring a negative power supply for the op-amp.

28.5 Peak detectors

The task of a peak detector circuit is to output a voltage proportional to the amplitude of the input signal. This has two main applications:

- Devices such as audio level meters where a light flashes when the incoming signal exceeds a threshold that will cause distortion later in the signal chain.
- Demodulators for amplitude modulated signals. (An amplitude modulated signal is one which conveys information by adjusting the amplitude of a sinewave *carrier*. To extract the information, a signal is required that can track the amplitude of the incoming waveform.)

This circuit differs from a rectifier since the maximum (or peak) value of input signal is ideally output for the whole of the period of the input waveform, rather than simply tracking the positive half-cycle as a rectifier would.

The simplest way to design a peak detector is to feed the output of a rectifier into a capacitor, with two resistors: one limiting the rate at which the capacitor can charge up and one limiting the rate at which the capacitor discharges, see Figure 28.15.

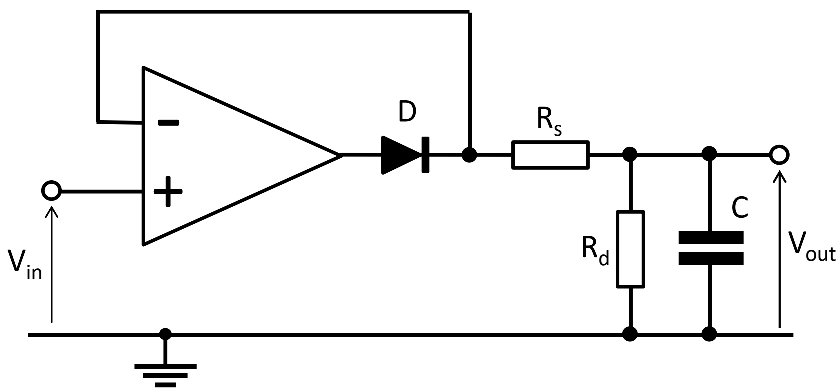

Figure 28.15 A typical peak-detector circuit

Note that this circuit deviates from the description of the ideal peak detector characteristic described above in two regards: if the input suddenly increases then the output won't suddenly increase, since the capacitor will take some time to charge up through the charging resistor R_s, and the peak value of the waveform is not held indefinitely: as soon as the input decreases from the peak voltage on the capacitor the capacitor will start to discharge through discharge resistor R_d.

This is what is required in both of the application of this circuit: for an audio-level meter you want the output to decay once the amplitude of the incoming signal decreases (so that you can see the effect of reducing the input amplitude), for demodulating an amplitude-modulated signal you want the input to continually adapt to the incoming signal.

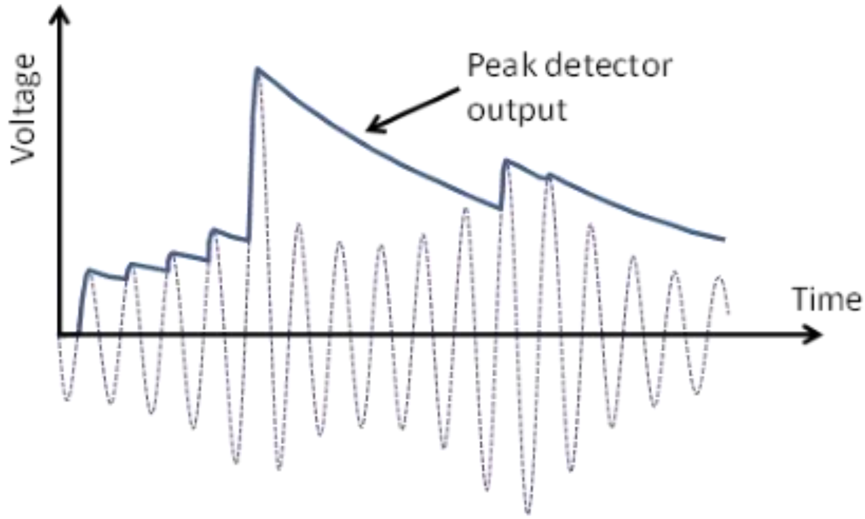

Figure 28.16 Input and output waveforms of a typical peak detector

The behaviour of a peak-detector is determined by the time-constants: the charging time constant and the decaying time constant.

The discharging time constant is easy to determine, since when the voltage on the capacitor is higher than the input voltage, the only way the charge on the capacitor can get to ground is through the discharging resistor[7]. This makes the time-constant of the discharge $t = C R_d$.

In most cases, the charging time-constant is much faster than the discharging time-constant, and in those cases the charging time-constant can be well-approximated by $t = C R_s$. Note however that this is not exact, since some of the current flowing through the charging resistor will not flow onto the capacitor, but instead will flow through the discharging resistor to ground. A good way to analyse this circuit is to construct the Thévenin equivalent of the circuit driving the charging capacitor. (Unlike most circuits containing diodes this is possible in this case, since during the charging process the superdiode acts as a buffer, which is a linear circuit.)

This leads to an equivalent circuit connected to the capacitor as shown in Figure 28.17 and a charging time-constant of $t = C R_s R_d / (R_s + R_d)$. (Note also that this means the capacitor never charges up to Vin, but only to Thévenin equivalent voltage of this circuit.)

[7] At least neglecting the saturation current through the diode, the input bias currents to the op-amp and any current that flows out of the output into the next stage.

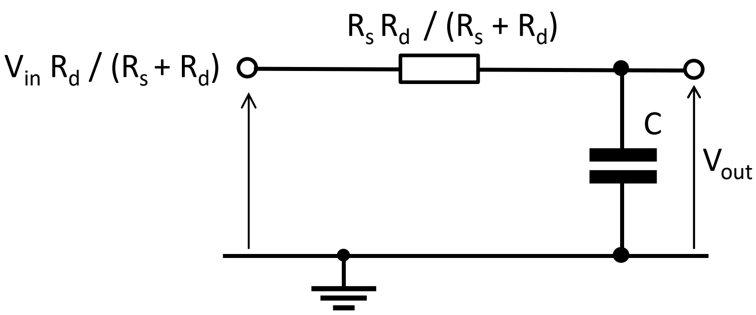

Figure 28.17 Equivalent circuit for charging the capacitor in a peak detector

28.6 Logarithms, exponentials, multipliers and dividers

Another set of useful op-amp circuits are those that have a logarithmic (or exponential) relationship between their inputs and outputs. Consider the circuit shown below with an input at A and an output at X:

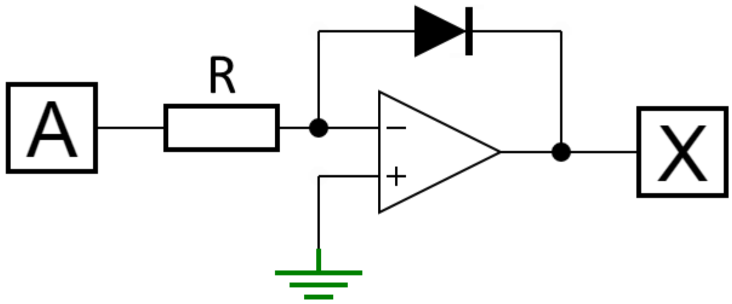

Figure 28.18 A log-amp

Consider a positive input voltage of *V* volts supplied at point A. The inverting input is held at ground by the feedback action of the op-amp, so the current flowing in through resistor *R* will be given by:

$$I = \frac{V}{R} \tag{28.10}$$

Using the Shockley equation:

$$I = I_s\left(\exp\left(\frac{eV_D}{nkT}\right) - 1\right) \tag{28.11}$$

the voltage across the diode when this current is flowing through it is given by:

$$V_D = \frac{nkT}{e}\ln\left(\frac{I}{I_S} + 1\right) \tag{28.12}$$

In the usual case where the current is much greater than the diode's saturation current I_s, this can be approximated as:

$$V_D = \frac{nkT}{e}\ln\left(\frac{I}{I_S}\right) = \frac{nkT}{e}\ln(I) - \frac{nkT}{e}\ln(I_S) \tag{28.13}$$

which, apart from the constant offset[8], leads to an output which is proportional to the logarithm of the input. This is known as a *logarithmic amplifier*, or *log-amp* for short.

This circuit only works for positive input voltages. If you were only interested in negative inputs you could just turn the diode round; for both positive and negative inputs, two diodes in parallel pointing in opposite directions gives an output proportional to the logarithm of the magnitude of the input, and opposite sign to the input.

We can go the other way as well:

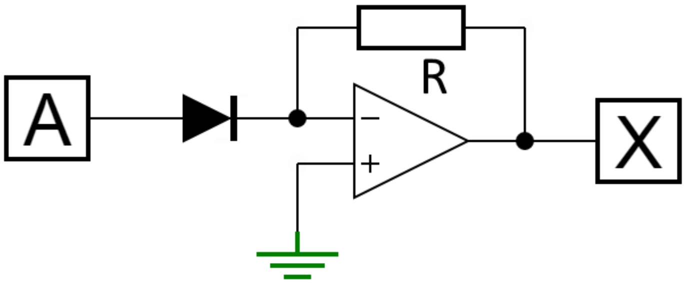

Figure 28.19 An exponential amplifier

In this case the output at X is given in terms of the input at A by:

$$V_{out} = -RI_S\left(\exp\left(\frac{eV_{in}}{nkT}\right)-1\right) \approx -RI_S\exp\left(\frac{eV_{in}}{nkT}\right) \tag{28.14}$$

With circuits that can take logs, and other circuits that can add two signals (a summing amplifier), and a circuit that can take exponentials, we can build a multiplier:

$$X \times Y = \exp\left(\log\left(X\right)+\log\left(Y\right)\right) \tag{28.15}$$

and with a differential amplifier that outputs the difference between two signals, we can build a divider:

$$\frac{X}{Y} = \exp\left(\log\left(X\right)-\log\left(Y\right)\right) \tag{28.16}$$

Welcome to the wonderful world of analogue computing.

28.7 Summary: the most important things to know

- The Schmitt trigger uses positive feedback to provide two switching thresholds (one for rising inputs, one for falling inputs) which helps it perform well with noisy inputs.
- Putting a diode in the feedback loop of an op-amp can produce a “superdiode” with no forward voltage drop.

[8] It’s worth noting that the constant offset is a function of the saturation current of the diode, and the saturation current of the diode is a strong function of temperature, so in practice some temperature-compensation is often required.

- Full-wave rectifiers produce an output equal to the modulus of the input voltage and can be built using op-amps with no voltage drop.
- Using diodes with op-amps can also produce circuits which output a voltage proportional to the logarithm or exponential of the input voltage.