27 A Short Introduction to Active Filters

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Prerequisite knowledge required: Op-Amps, Linear Op-Amp Circuits, AC Circuit Analysis

27.1 Introduction to filters

Filters are very useful circuits with a lot of different applications. For example, most analogue-to-digital converters (ADCs) will have an anti-aliasing filter before them which removes all frequencies above half the sampling rate of the converter. (If this is not done, then any signal at these higher frequencies will appear, after conversion, to be at a lower frequency than there are. See Figure 27.1 where a fast incoming signal (the blue sine wave) is sampled at regular intervals (the orange dots). The sampled values are exactly the same as would be produced by a much slower sine wave, which means that any digital signal processing occurring on the converted values could not tell the difference. The solution is to remove all of these unwanted high frequencies before the ADC using a filter.)

Since it's usually desirable not to introduce any distortion in the signal being converted, this suggests the use of a filter which has as flat a frequency response as possible over the frequencies of interest in the application, but which can remove all frequencies above half the sampling frequency to remove any ambiguity in the digitised waveform.

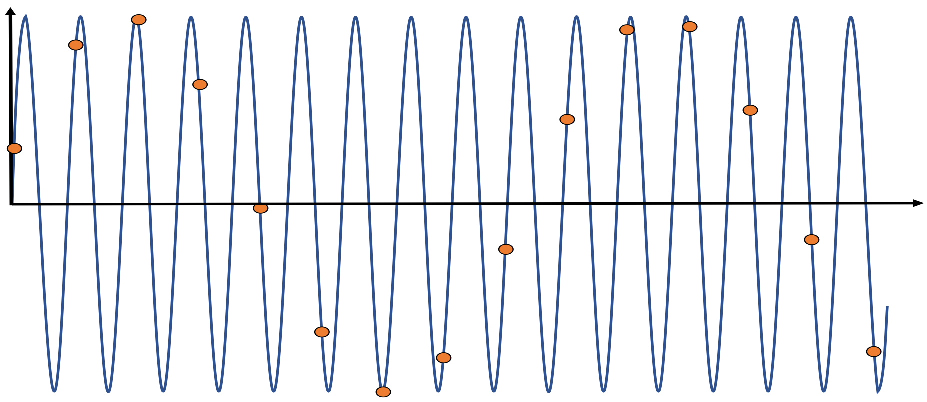

Figure 27.1 Illustration of the aliasing problem in analogue-to-digital conversion

The faster the sampling frequency, the more expensive and power-hungry the ADC, so what's required is a filter with a very sharp drop from the passband frequencies (those it is designed to let through) to the stopband frequencies (those it is designed to get rid of).

Another very different application of filters is in audio tone control circuits. Here what is required is a variable response, one that can be easily changed from amplifying to attenuating a particular range of frequencies. These require some very clever circuit configurations.

27.2 Why use active filters?

First, some definitions: a filter is a circuit designed to have different gains at different frequencies; an active filter is one which includes an active circuit element in the filter. For the purposes of this module, the only active components that we'll consider are op-amps.

The opposite of an active filter is a passive filter. Passive filters only contain passive components such as resistors, capacitors and inductors (all the filters we have seen so far have been passive filters). It's possible to build a filter with almost any frequency response just with passive components, and passive filters have several advantages over any circuit with active elements:

- Passive filters don't require power
- Passive filters don't distort large amplitude signals[1]
- Passive filters don't introduce as much noise[2]
- Passive filters don't suffer from any noise on the power supply coupling into the signal[3]

So why consider active filters at all? Well, they turn out to have a couple of key advantages:

- It's possible to design active filters with large Q-factors which do not require inductors (and inductors are large, expensive, difficult to use in integrated circuits and tend to introduce non-linearity)
- Active filters can be designed that only required variable resistors to control the filter characteristics in useful ways (rather than using more expensive variable capacitors or inductors)

In this note, I'll introduce a few of the most common and useful active filter circuits: the Baxandall tone control circuits, and two variations on the idea of a voltage-controlled voltage source (VCVS) filter.

27.3 The Baxandall tone control circuits

Many audio applications require circuits that can adjust the tone[4] of a signal, and it's useful to be able to design circuits which affect the tone in intuitive ways. The most common form of tone controls are the bass and treble controls present in a lot of hi-fi and stage amplification equipment.

A bass tone control increases (boosts) or decreases (cuts) the amplitudes of the low frequencies, leaving the higher frequencies unaffected. A treble tone control does the same job for the high frequencies, leaving the lower frequencies unaffected.

There are many circuits which can perform these functions, but one family of circuits which has stood the test of time was designed by Peter Baxandall in the early 1950s[5] [1]. The Baxandall tone control circuits are both simple and effective, and they have the key advantage that they can boost

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[1] Within reason, that is. The passive components will have maximum power ratings, and inductors can be non-linear. However active filters are restricted by the power supply voltages being used; for example op-amps can't output any voltage outside the range of their power supplies, and this introduces definite clipping and distortion at high signal amplitudes.

[2] Any active circuit element introduces additional noise into the signal path.

[3] Ideally, active filters would not do this either, but there is always a small breakthrough from noise on the power supply to the signal. This can be a particular problem when the incoming signal is small and due to be amplified by a large amount in the circuit, as positive feedback and instability can easily result.

[4] By 'tone' I mean the balance between the amplitudes of the low and high frequencies in a signal.

[5] P.J. Baxandall, "Negative Feedback Tone Control. Independent Variation of Bass and Treble Without Switches", Wireless World, Vol.58, No.10 (Oct. 1952) pp. 402. (Correction Vol.58, No.11 (Nov. 1952) pp. 444.)

or cut a range of frequencies using only a single potentiometer. (Potentiometers are cheaper and much more readily available than variable capacitors or variable inductors.)

There are a range of Baxandall circuits, here I'll just introduce the treble and bass controls. (Others exist which can control only the mid-range frequencies; there are also versions of the circuit which can combine treble and bass tone control circuits using a single op-amp. In fact the original circuit Baxandall proposed was a joint bass and treble tone control.)

27.3.1 The Baxandall treble tone control

Figure 27.2 below shows an op-amp circuit configured as a Baxandall treble tone control. Note that the op-amp in this circuit is being operated in inverting mode, so the output will go up when the input goes down and vice-versa[6].

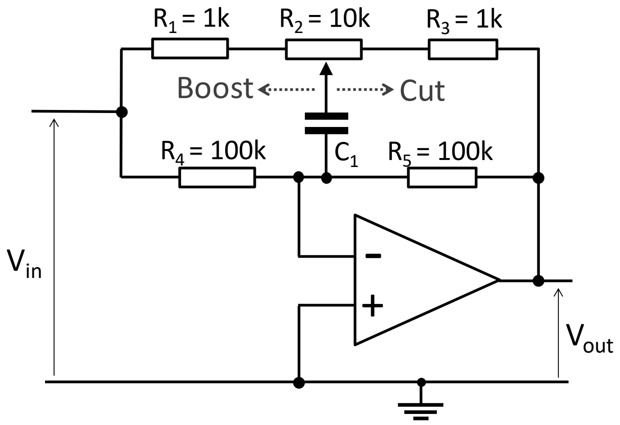

Figure 27.2 A Baxandall treble tone control circuit

27.3.1.1 The Baxandall treble tone control at low frequencies

To understand the operation of this circuit it's helpful to consider its operation at the extremes of frequency. At very low frequencies it's safe to assume that the capacitor has a very large impedance, much larger than the resistors, and therefore it plays no part in the circuit. This reduces the circuit to:

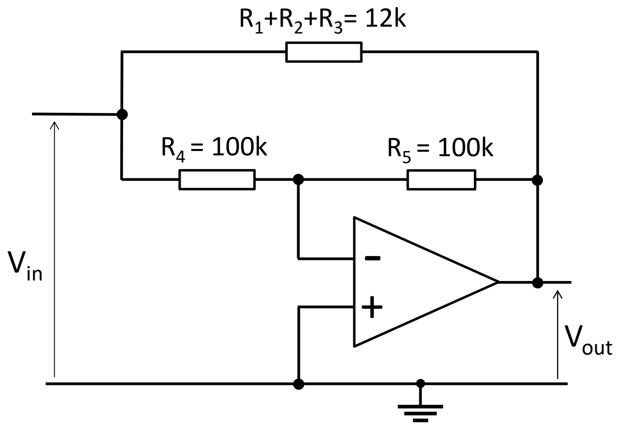

Figure 27.3 A Baxandall treble tone control circuit at very low frequencies

[6] This isn't usually a problem in audio applications.

and this is just an inverting amplifier with a gain of minus one at all frequencies, with an additional 12k resistor between the input and the output (which doesn't affect the gain of the circuit at all, it just causes a bit more current to flow from the input to the output).

27.3.1.2 The Baxandall treble tone control with the potentiometer set mid-way

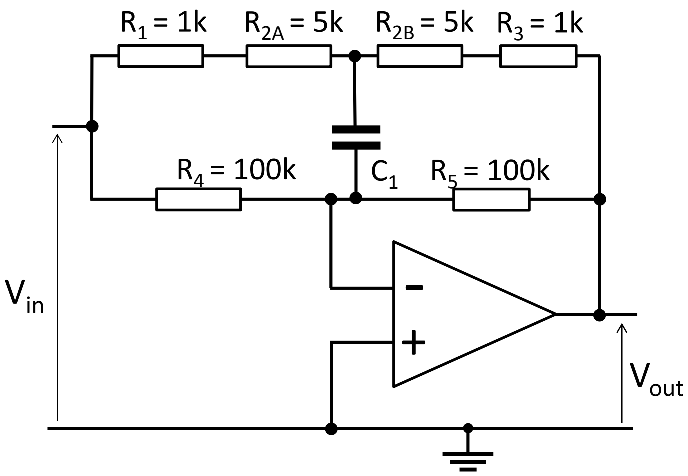

Figure 27.4 Baxandall treble tone control with no cut or boost

Another special case can be considered in an intuitive way: the case where the potentiometer is set mid-way, so that the circuit becomes that shown in Figure 27.4.

In this case, consider the circuit nodes at both ends of the capacitor. If the capacitor was not there, the upper node would be at voltage half-way between V_{in} and V_{out} (since it would be in the middle of a potential divider with 1k + 5k = 6k to each side). The lower terminal would also be at a voltage half-way between V_{in} and V_{out} (since it would be in the middle of a potential divider with 100k to each side). So if the capacitor was introduced into the circuit, no current would flow through it, since the voltage on both ends would be the same. In other words the capacitor might as well not be there, it plays no part in setting the voltages at any node in the circuit.

Without the capacitor, this circuit behaves just like it would at very low frequencies where the capacitor has a very large impedance, and we've just seen that that implies the gain of the circuit is minus one for all frequencies.

27.3.1.3 The Baxandall treble tone control at high frequencies

However at very high frequencies, so high that the capacitor has a much lower impedance than the resistors and can be approximated by a wire, the equivalent circuit becomes:

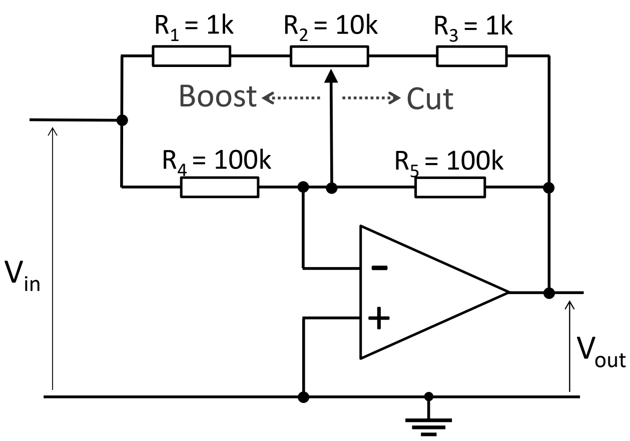

Figure 27.5 A Baxandall treble tone control circuit at very high frequencies

At this point we could make an approximation[7], and consider that the 100k resistors R4 and R5 are sufficiently larger than the values of the resistors R1, R2 and R3 that their contribution to the performance of the circuit can be neglected, and therefore simply this still further to:

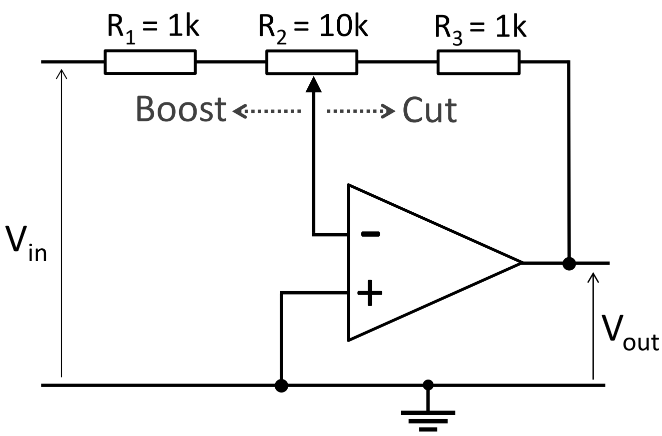

Figure 27.6 A Baxandall treble tone control circuit at very high frequencies

which is perhaps the most intuitive way to see what is going on. With the potentiometer moved fully towards the "Boost" position, this produces an inverting amplifier with a resistor of 1k between V_{in} and the inverting input of the op-amp, and of 1k + 10k = 11k between the inverting input and the output of the op-amp. This produces an amplifier with a voltage gain of:

$$G(j\omega) = \frac{\mathbf{V_{out}}}{\mathbf{V_{in}}} = -\frac{11\text{ k}}{1\text{ k}} = -11 \qquad (27.1)$$

so these high-frequencies will be increased in amplitude by a factor of eleven, independent of their exact frequency. On the other hand, move the potentiometer fully towards the "Cut" position, and the resistance between V_{in} and the op-amp's inverting input becomes 1k + 10k = 11k and the

[7] This isn't an entirely accurate approximation, and a better analysis including the effects of R_4 and R_5 reveals that the actual gain at high-frequencies in the full-boost position is -1111/111 which is almost exactly -10, and in the full-cut position is -111/1111 which is almost exactly -0.1.

feedback resistance between the op-amp's output and inverting input becomes 1k, so the voltage gain is now:

$$G(j\omega)=\frac{\mathbf{V}_{\mathbf{out}}}{\mathbf{V}_{\mathbf{in}}}=-\frac{1\text{ k}}{11\text{ k}}=-\frac{1}{11} \tag{27.2}$$

So what we have is a circuit which does not affect low frequencies at all, but which can either amplify or attenuate high-frequencies. Exactly what is required for a treble tone control.

27.3.1.4 What about the poles and zeros?

To work out what happens for frequencies in-between the very low (for which we can assume the capacitor is an open-circuit) and very high (for which we can assume the capacitor is a short circuit), we need to do a more careful analysis.

Assuming the op-amp is ideal, we can solve the circuit shown in Figure 27.2 **Error! Reference source not found.**using standard circuit analysis techniques. The derivation is a little tedious, but eventually gives:

$$G(j\omega)=\frac{\mathbf{V}_{\mathbf{out}}}{\mathbf{V}_{\mathbf{in}}}=-\frac{R_5}{R_4}\frac{1+j\omega\dfrac{CR_4\left(R_3+\alpha R_2\right)}{R_1+R_2+R_3}}{1+j\omega\dfrac{CR_5\left(R_1+(1-\alpha)R_2\right)}{R_1+R_2+R_3}} \tag{27.3}$$

where the relative position of the potentiometer is given in terms of *α*, where *α* = 1 means the circuit is set for full boost, and *α* = 0 means it is set for full cut.

When $R_4 = R_5$ and $R_1 = R_3$, this reveals a low-frequency gain of minus one, a zero with a break frequency at:

$$\omega_Z=\frac{2R_1+R_2}{CR_4\left(R_1+\alpha R_2\right)} \tag{27.4}$$

and a pole with a break frequency at:

$$\omega_P=\frac{2R_1+R_2}{CR_4\left(R_1+(1-\alpha)R_2\right)} \tag{27.5}$$

Plotting the break frequency of this pole and zero for typical resistance values (those shown in Figure 27.2 with a capacitor of 10 nF) against *α* gives:

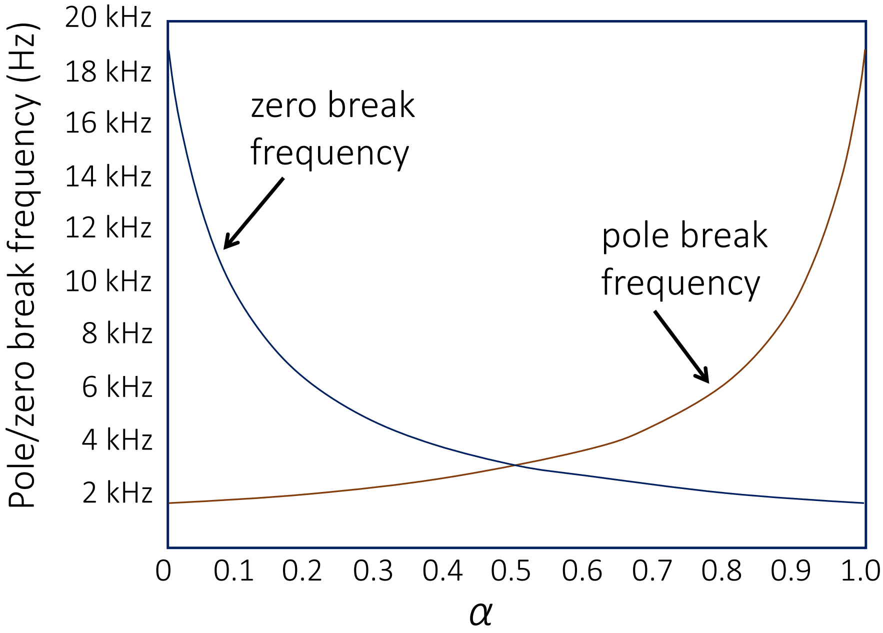

Figure 27.7 Movement of the pole and zero break frequencies in a Baxandall treble tone control

You can see that when $\alpha = 0.5$ (mid-way on the potentiometer), the break frequencies of the pole and the zero are equal, so they cancel each other out, and the result is a flat frequency response.

When α is zero, the pole's break frequency happens at around 1.9 kHz, so this is where the gain of the circuit starts to drop significantly. It continues to drop until approaching the zero's break frequency at around 19 kHz, where it levels off: this provides a cut of treble frequencies. At the other extreme when α is one, the zero's break frequency happens first (at around 1.9 kHz), so the gain of the circuit first starts to increase at around 20 dB/decade until it approaches the pole's break frequency (now at around 19 kHz) where it levels off: this provides a boost in the treble frequencies.

A plot of the frequency response for a few different values of α is shown below:

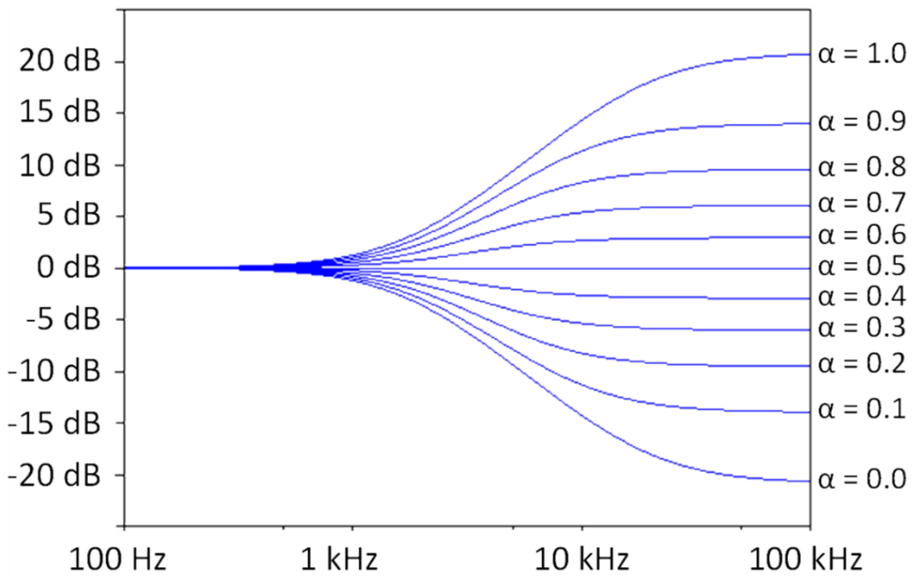

Figure 27.8 A Baxandall treble tone control amplitude response for various potentiometer positions

Note that with a factor of 10 between the zero and the pole break frequencies in both extreme cases, the boost or cut at the higher frequencies must be 20 dB (the Bode approximation technique tells us that a good approximate to the frequency response can be obtained by drawing a line at 20 dB/decade between the first turning point and the second, and a factor of 10 in frequency makes one decade, so a total difference in levels between low and high frequencies of 20 dB).

For intermediate values of α there is less than a decade's worth of frequencies between the two break frequencies, and therefore less than 20 dB difference between the low and high frequency gains.

27.3.2 A Baxandall bass tone control circuit

Baxandall also described the design of a bass tone control. It operates on similar principles to the treble tone control circuit discussed in the previous section, but with a slightly different circuit arrangement (see Figure 27.9).

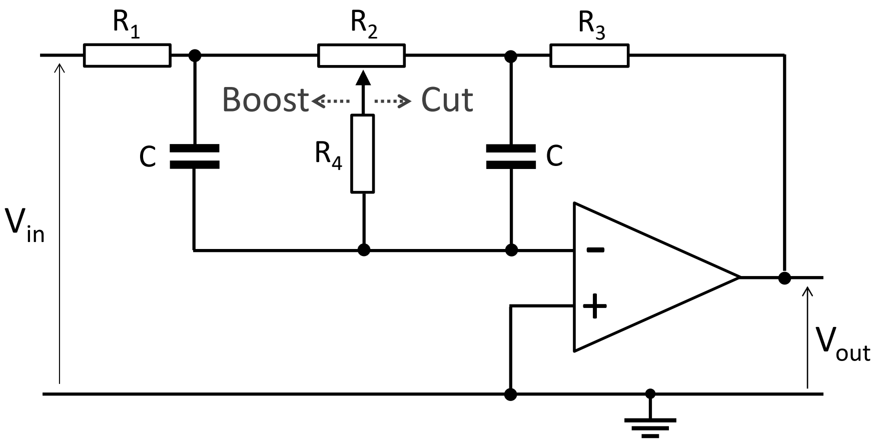

Figure 27.9 A Baxandall bass tone control circuit

It can be analysed in a similar way: at high frequencies the low impedances of the capacitors result in no current flowing through the potentiometer at all (it all flows around though the two capacitors), and so the position of the potentiometer is irrelevant: both ends of the potentiometer are at the same voltage. The circuit effectively behaves as shown in Figure 27.10, which has a voltage gain of minus one.

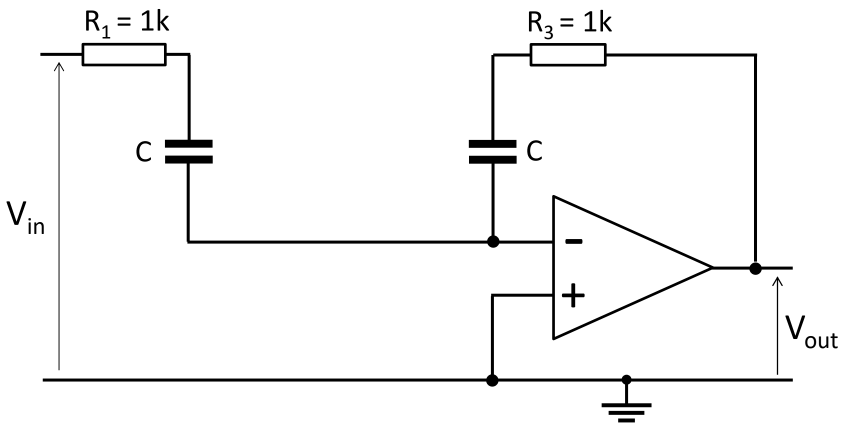

Figure 27.10 A Baxandall bass tone control at high frequencies

At very low frequencies, where the capacitors effectively behave as open-circuits, the circuit becomes:

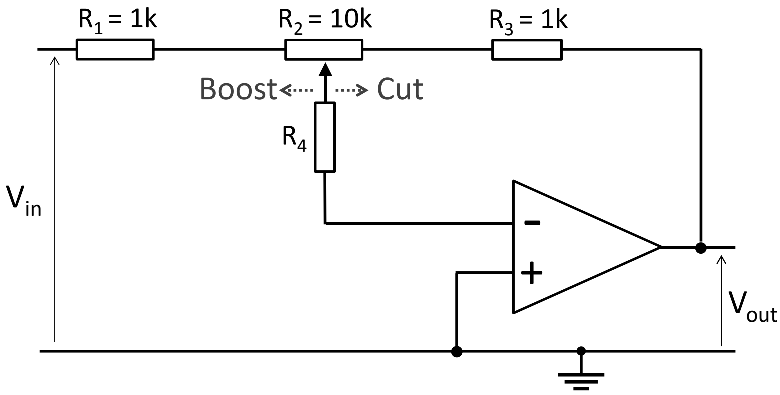

Figure 27.11 A Baxandall bass tone control at very low frequencies

which with these circuit values has a gain variable between 11 and 1/11 (depending on the position of the potentiometer[8]).

In-between, a similar analysis to that done for the treble tone control reveals a single dominant pole and zero again, with break frequencies that move up and down in frequency in opposite directions, and coincide when the potentiometer is placed at its mid-position[9]. The derivation is rather long-winded, so I'll leave that as an exercise to the careful reader with a lot of spare time[10], but if you assume that the resistor R_4 is much greater than the other resistors (a common case in practice), then the gain of the circuit can be approximated by:

$$G(j\omega) = \frac{\mathbf{V}_{\mathbf{out}}}{\mathbf{V}_{\mathbf{in}}} \approx -\frac{\left(R_1 + (1-\alpha)R_2\right)}{\left(R_1 + \alpha R_2\right)} \frac{\left(1 + \dfrac{j\omega C R_4 \left(2R_1 + R_2\right)}{\left(R_1 + \alpha R_2\right)}\right)}{\left(1 + \dfrac{j\omega C R_4 \left(2R_1 + R_2\right)}{\left(R_1 + (1-\alpha)R_2\right)}\right)} \tag{27.6}$$

which reveals a pole and zero with break frequencies at:

$$\begin{aligned} \omega_p &= \frac{R + (1-\alpha)R_2}{CR_4\left(2R_1 + R_2\right)} \\ \omega_z &= \frac{R + \alpha R_2}{CR_4\left(2R_1 + R_2\right)} \end{aligned} \tag{27.7}$$

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[8] This time this is a good approximation: the gain at very low frequencies really does vary between -11 (20.8 dB) and -1/11 (-20.8 dB).

[9] I've had to say "dominant pole and zero" in this case since a full analysis of the circuit reveals that there are two poles and two zeros; however provided R_4 is chosen to be larger than the other resistors in the circuit, the additional pole and zero tend to cancel each other out, and the frequency response of the whole circuit is very well approximated by a single pole and zero.

[10] I'll be impressed if anyone manages to do this. Hint: try using the Δ-Y transform on the RC network.

As expected, when $\alpha = 0.5$ the pole and zero coincide and cancel out, and when α increases, the pole's break frequency decreases while the zero's break frequency increases.

The overall frequency responses for this circuit for different values of α look something like:

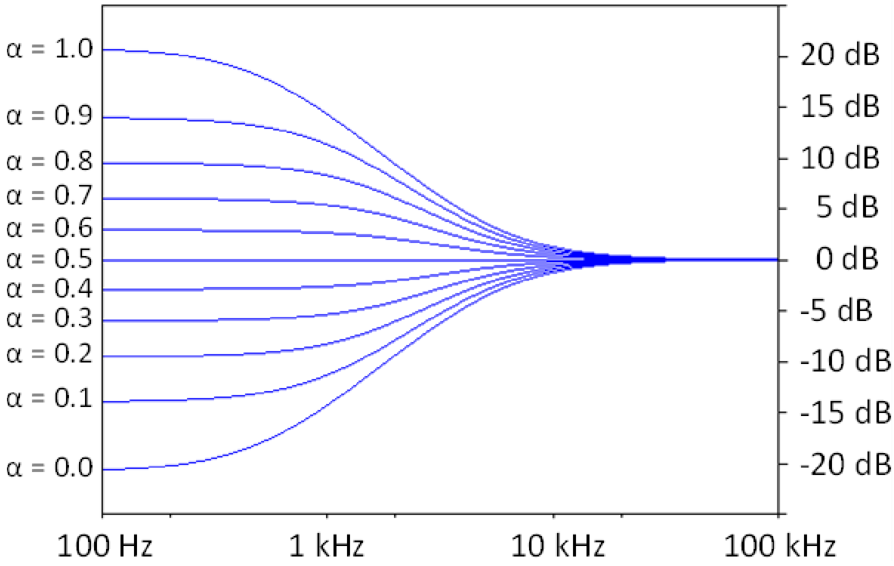

Figure 27.12 A Baxandall bass tone control amplitude response for various potentiometer positions

27.3.3 Another Baxandall bass tone control circuit

There's a cheaper way to make a Baxandall bass tone control, and one that is easier to analyse, since it only uses one capacitor. The circuit is shown in Figure 27.13.

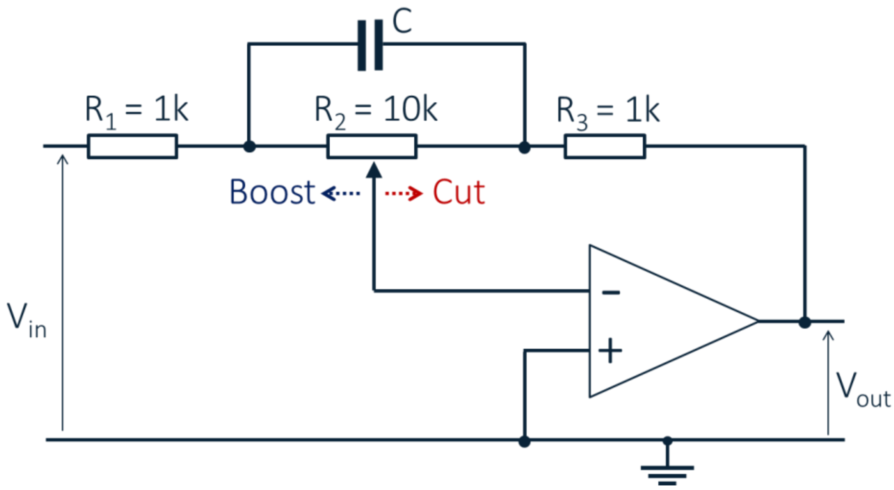

Figure 27.13 A Baxandall bass tone control with only one capacitor

Analysis of this circuit reveals a gain of:

$$G(j\omega) = \frac{\mathbf{V}_{\mathbf{out}}}{\mathbf{V}_{\mathbf{in}}} \approx -\frac{\left(R_3 + (1-\alpha)R_2\right)\left(1 + j\omega C \dfrac{R_2 R_3}{R_3 + (1-\alpha)R_2}\right)}{\left(R_1 + \alpha R_2\right)\left(1 + j\omega C \dfrac{R_1 R_2}{R_1 + \alpha R_2}\right)} \tag{27.8}$$

and this also gives a maximum gain of 11 and minimum gain of 1/11, while leaving the high frequencies unaffected. (Proof of this is left as an exercise for the reader, it goes along the same lines as the previous derivation: assume the capacitor can be replaced by a wire at very high frequency, and by an open circuit at very low frequencies).

27.4 The combined bass and treble Baxandall tone control

Most often you don't want just a bass or just a treble tone control, you want both. There are several designs based on the Baxandall circuits which can provide this. For example, consider the circuit shown below:

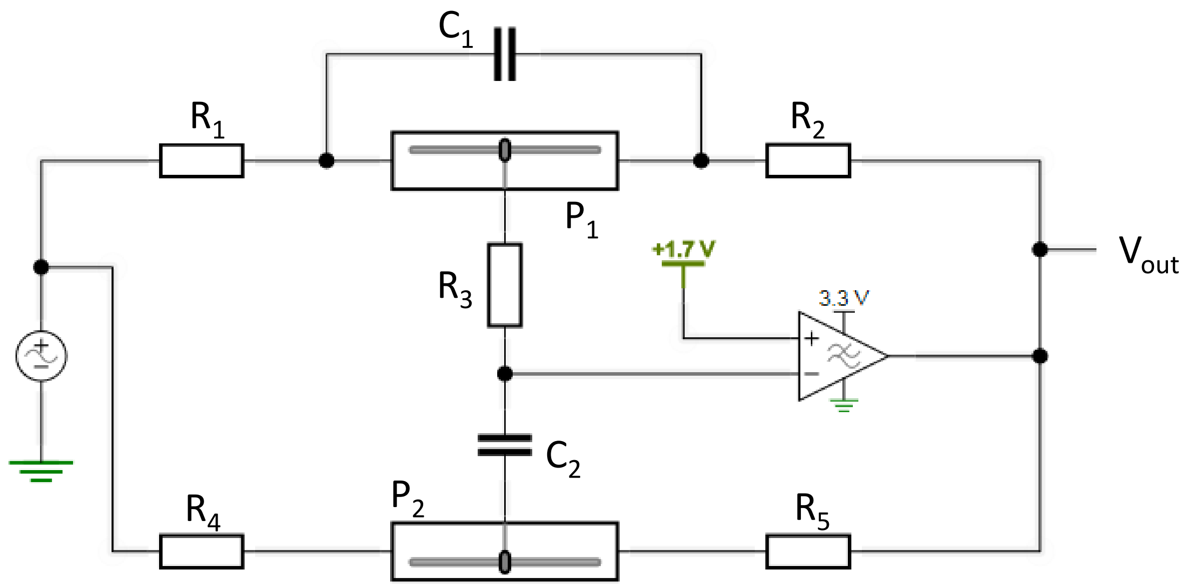

Figure 27.14 The combined bass-treble Baxandall circuit used on the VAM

A simple approximate analysis of this circuit can be done by making the assumption that the bass and treble controls are independent. For bass frequencies, this would mean that the smaller capacitor C_2 is assumed to be so small that its impedance is much greater than the resistor R_3, and therefore that the voltage on the inverting input to the op-amp is dominated by the voltage from the potentiometer P_1, which varies with low frequencies, but not at high frequencies (since at high frequencies the voltage at both sides of P_1, and therefore at all points within P_1, is held constant by capacitor C_1).

For treble frequencies, the opposite assumption is made: that the impedance of C_2 is much less than than of R_3, so the inverting input to the op-amp is determined by the voltage from the lower potentiometer, all the negative feedback into the op-amp comes through the capacitor C_2 which now has a much smaller impedance than the resistor R_2.

The full analysis required to determine the break frequencies of the poles and zeros is rather complex, since there is an interaction between the bass and treble tone controls. In general, optimising the values of the capacitors is usually easier done by simulation.

While the Baxandall circuits are very useful in giving controllable responses, they only provide a single (usable) pole and zero, and therefore cannot reproduce the resonant behaviour characteristic of passive LCR circuits. There are however other active filter circuits that can do this, read on for more details.

27.5 The voltage-controlled voltage source (VCVS) filter

The voltage-controlled voltage source (VCVS) filter circuit is a useful filter circuit, since it can simulate the resonant behaviour of an LCR circuit without requiring a large expensive inductor, and is comparatively simple to analyse. It also has the additional benefit that the resultant circuit can have voltage gain at low frequencies. The main disadvantages are the usual ones for active filters: limited dynamic range, additional noise and the requirement for a power supply, and also for this circuit in particular, that it has a fixed, defined relationship between the gain and the Q-factor: you cannot set one independently of the other, and this limits the circuit's usefulness in some circumstances.

The circuit looks like this:

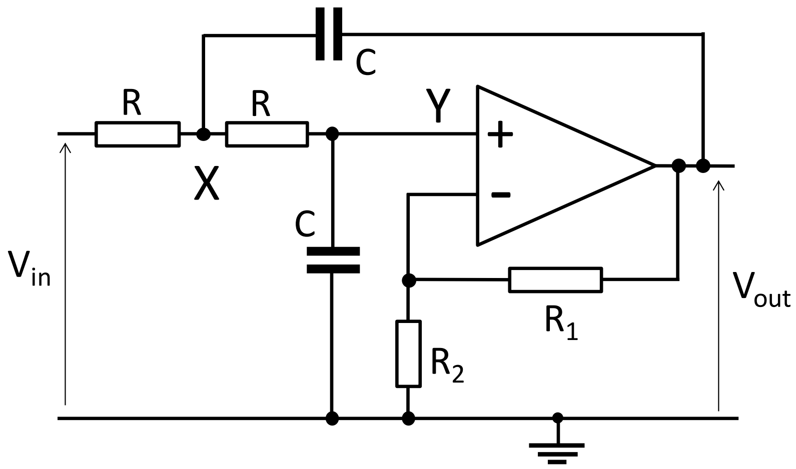

Figure 27.15 The voltage-controlled voltage source active filter circuit

Analysis is straightforward, if a little tedious. Applying Kirchhoff's and Ohm's laws to the circuit above produces the following equations (assuming an ideal op-amp):

For currents flowing into node X (where **X** is the phasor representation of the voltage at node X, and **Y** is the phasor representation of the voltage at node Y):

$$\frac{(\mathbf{V}_{in} - \mathbf{X})}{R} + \frac{(\mathbf{V}_{out} - \mathbf{X})}{1/j\omega C} = \frac{(\mathbf{X} - \mathbf{Y})}{R} \tag{27.9}$$

For the potential divider between X, Y and ground:

$$\mathbf{Y} = \mathbf{X}\frac{1/j\omega C}{R + 1/j\omega C} \tag{27.10}$$

For the potential divider generating the feedback signal, noting that for stable operation with an ideal op-amp the voltages at the inverting input and non-inverting input must be equal:

$$\mathbf{Y} = \mathbf{V}_{out}\frac{R_2}{R_1 + R_2} \tag{27.11}$$

That's three equations in three unknowns (**X**, **Y** and $\mathbf{V}_{out}$), so that calls for some tedious algebra. First, eliminate Y between (27.11) and (27.10):

$$\mathbf{X} = \mathbf{V}_{out} \frac{R_2\left(1+j\omega RC\right)}{R_1+R_2} \tag{27.12}$$

Then using (27.12) and (27.11) to eliminate **X** and **Y** from (27.9):

$$\frac{\mathbf{V}_{in}}{R} - \mathbf{V}_{out}\frac{R_2\left(1+j\omega RC\right)}{R\left(R_1+R_2\right)} + j\omega C\,\mathbf{V}_{out} - \mathbf{V}_{out}\frac{R_2\left(1+j\omega RC\right)j\omega C}{R_1+R_2} = \mathbf{V}_{out}\frac{R_2\left(1+j\omega RC\right)}{R\left(R_1+R_2\right)} - \mathbf{V}_{out}\frac{R_2}{R\left(R_1+R_2\right)} \tag{27.13}$$

Collecting terms in V_{in} and V_{out}:

$$\mathbf{V}_{out}\frac{R_2\left(1+j\omega RC\right)}{R\left(R_1+R_2\right)} - \mathbf{V}_{out}\frac{R_2}{R\left(R_1+R_2\right)} + \mathbf{V}_{out}\frac{R_2\left(1+j\omega RC\right)}{R\left(R_1+R_2\right)} - j\omega C\,\mathbf{V}_{out} + \mathbf{V}_{out}\frac{R_2\left(1+j\omega RC\right)j\omega C}{R_1+R_2} = \frac{\mathbf{V}_{in}}{R} \tag{27.14}$$

and simplifying:

$$\begin{aligned}&\mathbf{V}_{out}R_2\left(1+j\omega RC\right) - \mathbf{V}_{out}R_2 + \mathbf{V}_{out}R_2\left(1+j\omega RC\right) \\ &\quad - j\omega RC\left(R_1+R_2\right)\mathbf{V}_{out} + \mathbf{V}_{out}R_2R\left(1+j\omega RC\right)j\omega C = \mathbf{V}_{in}\left(R_1+R_2\right)\end{aligned} \tag{27.15}$$

$$\begin{aligned}&\mathbf{V}_{out}R_2 + 3\mathbf{V}_{out}R_2 j\omega RC - j\omega RC\left(R_1+R_2\right)\mathbf{V}_{out} \\ &\qquad - \mathbf{V}_{out}R_2R^2\omega^2C^2 = \mathbf{V}_{in}\left(R_1+R_2\right)\end{aligned} \tag{27.16}$$

$$\mathbf{V}_{out}\left(R_2 + 3R_2 j\omega RC - j\omega RC\left(R_1+R_2\right) - R_2R^2\omega^2C^2\right) = \mathbf{V}_{in}\left(R_1+R_2\right) \tag{27.17}$$

$$\frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{R_1+R_2}{R_2 + 3R_2 j\omega RC - j\omega RC\left(R_1+R_2\right) - R_2R^2\omega^2C^2} \tag{27.18}$$

Dividing through by R_2 gives:

$$\frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{\left(\dfrac{R_1+R_2}{R_2}\right)}{1 + j\omega RC\left(3 - \dfrac{R_1+R_2}{R_2}\right) - \omega^2R^2C^2} \tag{27.19}$$

and comparing this with the general form for a second-order response:

$$\frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{K}{1 + \dfrac{j\omega}{Q\omega_0} - \dfrac{\omega^2}{{\omega_0}^2}} \tag{27.20}$$

reveals that this circuit does indeed have the form of a second-order response, with:

$$\omega_0 = \frac{1}{RC} \qquad \text{and} \qquad Q = \frac{1}{3 - \dfrac{R_1+R_2}{R_2}} \tag{27.21}$$

and a low-frequency gain of:

$$G_{DC} = K = \left(\frac{R_1 + R_2}{R_2} \right) \tag{27.22}$$

So we could write the Q-factor as:

$$Q = \frac{1}{3 - K} = \frac{1}{3 - G_{DC}} \tag{27.23}$$

where G_{DC} is the gain at low frequencies. This is the main problem with this circuit: you cannot set the gain and the Q-factor independently. There are some other active filter circuits that do not have this limitation.

27.6 The VAM filter circuit

An active filter circuit often used in audio applications is a slight variation on the voltage-controlled voltage source (VCVS) filter circuit described above, based on an inverting op-amp configuration. (This one is sometimes called a *multiple feedback* filter since there are two components that provide feedback from the output to different parts of the circuit.)

Just like the VCVS filter it can simulate the behaviour of a resonant LCR circuit without requiring a large expensive inductor and again it does this by using an op-amp and a second capacitor instead.

However this circuit has one key advantage over the previous circuit: the Q-factor can be set independently of the gain, and this is very useful in a circuit which requires no amplification (for this application the magnitude of the voltage gain at low frequencies has to be one). The circuit is however slightly more difficult to design, since the resultant expressions for the resonant frequency and Q-factor are a bit more complex.

The circuit looks like this:

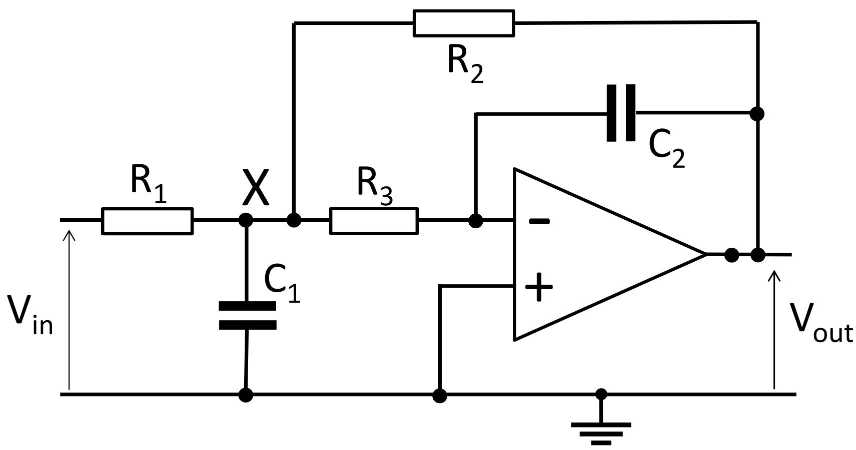

Figure 27.16 The active filter used on the VAM soundcard inputs and outputs

Like any of these circuits it can be analysed using straightforward circuit analysis techniques.

Applying Kirchhoff's and Ohm's laws to the circuit above produces the following equations (assuming an ideal op-amp, where the voltage difference between the inverting and non-inverting inputs is so small that the inverting input can be considered to be at ground):

For currents flowing into the junction marked with an X (**X** is the phasor representation of the voltage at this node):

$$\frac{(\mathbf{V_{in}} - \mathbf{X})}{R_1} = \frac{\mathbf{X}}{1/j\omega C_1} + \frac{(\mathbf{X} - \mathbf{V_{out}})}{R_2} + \frac{\mathbf{X}}{R_3} \qquad (27.24)$$

Also, the current flowing through R_3 and C_2 must be the same (assuming an ideal op-amp with no input current):

$$\frac{\mathbf{X}}{R_3} = -\frac{\mathbf{V_{out}}}{1/j\omega C_2} \qquad (27.25)$$

Substituting equation (27.25) into equation (27.24) to eliminate X gives:

$$\begin{aligned}\frac{\mathbf{V_{in}}}{R_1} &= X\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + j\omega C_1\right) - \frac{\mathbf{V_{out}}}{R_2} \\ &= -\mathbf{V_{out}} R_3 j\omega C_2 \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + j\omega C_1\right) - \frac{\mathbf{V_{out}}}{R_2}\end{aligned} \qquad (27.26)$$

which when re-arranged gives:

$$\begin{aligned}\frac{\mathbf{V_{out}}}{\mathbf{V_{in}}} &= \frac{-1/R_1}{\left(\frac{1}{R_2} + j\omega C_2 + j\omega \frac{R_3}{R_1} C_2 + j\omega \frac{R_3}{R_2} C_2 - \omega^2 C_1 C_2 R_3\right)} \\ &= \frac{-R_2/R_1}{\left(1 + j\omega C_2 R_2 + j\omega \frac{R_2 R_3}{R_1} C_2 + j\omega R_3 C_2 - \omega^2 C_1 C_2 R_2 R_3\right)}\end{aligned} \qquad (27.27)$$

Comparing this with the general form for a second-order response:

$$\frac{\mathbf{V_{out}}}{\mathbf{V_{in}}} = \frac{K}{1 + \frac{j\omega}{Q\omega_0} - \frac{\omega^2}{\omega_0^2}} \qquad (27.28)$$

reveals that this circuit has the form of a second-order response, this time with:

$$\omega_0 = \sqrt{\frac{1}{R_1 R_3 C_1 C_2}} \quad \text{and} \qquad Q = \frac{\sqrt{\frac{R_2 R_3 C_1}{C_2}}}{R_2 + \frac{R_2 R_3}{R_1} + R_3} \qquad (27.29)$$

For typical values ($R_1 = R_2 = 12k$, $R_3 = 3k9$, $C_1 = 1n8$, $C_2 = 330p$), this results in a low-frequency gain of minus one, and a resonant frequency and Q-factor of:

$$\omega_0 = \sqrt{\frac{1}{12\times10^3 \times 3.9\times10^3 \times 1.8\times10^{-9} \times 330\times10^{-12}}} = \sqrt{3.60\times10^{10}} = 189.6 \text{ krad/s}$$

$$f_0 = \frac{\omega_0}{2\pi} = 30.2 \text{ kHz} \tag{27.30}$$

$$Q = \frac{\sqrt{12\times10^3 \times 3.9\times10^3 \times 1.8\times10^{-9} / 330\times10^{-12}}}{12\times10^3 + 3.9\times10^3 + 3.9\times10^3} = \frac{\sqrt{255.3\times10^6}}{19800} = 0.8 \tag{27.31}$$

A filter with a Q-factor of 0.8 has a (very) small resonant peak in the frequency response, and with cut-off frequency of 30.2 kHz the maximum gain of this filter is around +0.25 dB at 14.6 kHz. This is about as close to the ideal situation (no resonant peak while maximising the flatness of the gain of the filter up to the cut-off frequency) as can be achieved using standard E12 series components and is ideal for use as an anti-aliasing or reconstruction filter for audio signals.

27.7 Summary: the most important things to know

- Active filters have several advantages over passive filters:
    - They can produce complex poles without using inductors
    - They can include amplification in the filter
- … and a few disadvantages:
    - They require a power supply
    - They limit the maximum signal amplitude
    - They introduce noise
- VCVS (voltage-controlled voltage source) filters can be readily designed with any desired value of gain, resonant frequency and Q-factor
- Baxandall tone controls can adjust the bass or treble frequencies in a signal using only one capacitor as a reactive element, and a potentiometer to control the response.