17 A Short Introduction to Time Variance and Time Constants

v1.4 – June 2021

Prerequisite knowledge required: Electrical Concepts and Parameters, DC Circuit Analysis

17.1 Introduction

Up until now, we've mostly been considering circuits in which the voltages and currents are not functions of time. While that's a good way to start learning about circuits, it severely restricts the circuits we can analyse: most circuits that do anything useful have voltages and currents which do change with time. This chapter is a short introduction to some of these circuits, in particular those featuring a single capacitor or a single inductor.

As we've seen before, the equation defining a capacitance is:

$$Q = CV \tag{17.1}$$

where Q is the charge on one plate of a capacitor, C is the capacitance (which we can assume is constant), and V is the potential difference between the plates of the capacitor. For the capacitor to charge, a current must flow onto (or off from) the plates of the capacitor, and current is the rate at which charge moves, so we can differentiate this equation to get:

$$I = \frac{dQ}{dt} = C\frac{dV}{dt} \tag{17.2}$$

where *I* is the current flowing onto one plate (and off from the other plate) of the capacitor.

For inductors, the inductance is defined by the equation:

$$e = -L\frac{dI}{dt} \tag{17.3}$$

where *e* is a voltage difference generated in the opposite direction to the rate of change of the current.

As you can see, both equation (17.2) and equation (17.3) are functions of time.

Before leaving the introduction, a couple of very useful thing to note about these equations:

- You can't suddenly change the voltage across a capacitor. (It would take an infinite current to deliver the charge required in zero time, and that's impossible.)
- You can't suddenly change the current through an inductor. (From equation (17.3) that would require an infinite voltage to be generated.)

In both cases it takes some time for the circuit to react to any changes in voltages or currents imposed by the outside world, and the *time constant* of these circuits is a measure of how quickly the circuit can react. Time constants are times (and hence measured in seconds) and constants (and hence remain at the same value in any given circuit, no matter what currents are flowing and voltages exist around the circuit).

This chapter describes how these time constants arise, how to calculate them, and how to use them to predict the behaviour of circuits.

17.2 Charging a capacitor from a current source

The simplest time-varying circuit to analyse is that of a constant current source feeding a capacitor. The circuit would look like this:

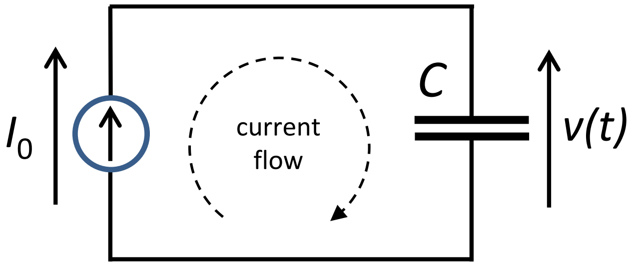

Figure 17.1 A capacitor charged from a constant current source

Assuming that the capacitor is uncharged at time t = 0, the voltage across the capacitor as a function of time can be derived from equation (17.2):

$$I_0 = C\frac{dV}{dt}$$
$$V(t) = \frac{1}{C}\int_0^t I_0\, dt = \frac{I_0}{C}\int_0^t dt = \frac{I_0}{C}[t]_0^t = \frac{I_0 t}{C} \qquad (17.4)$$

Plotting the voltage on the capacitor against time therefore gives a straight-line passing through zero, with a gradient of I/C, which looks like this:

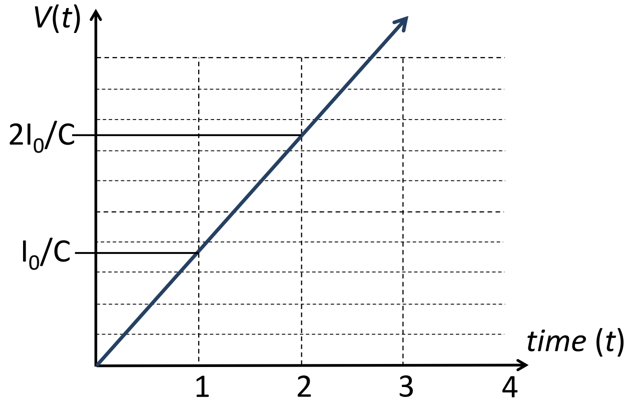

Figure 17.2 The voltage across a capacitor being charged by a constant current source

Of course, if the current is left on at the same constant value in any real circuit, at some point the voltage will exceed the voltage rating of the current source or the capacitor, but in theory, with ideal components, the voltage across the capacitor will just keep increasing without limit.

17.3 Charging an inductor from a voltage source

Only slightly more difficult to work out is what happens when you connect a constant voltage source to an inductor, as shown in Figure 17.3:

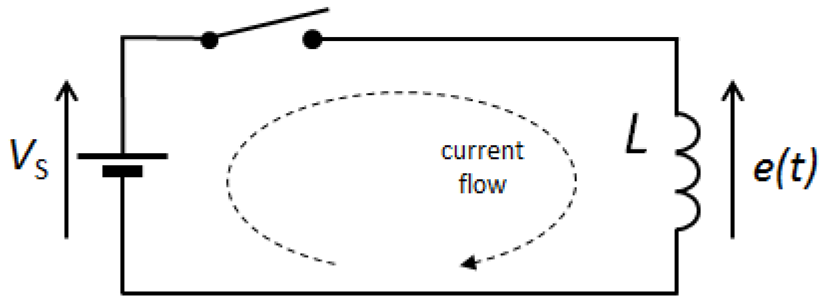

Figure 17.3 An inductor charging from a voltage source

When the switch is closed, current will start to try and flow around the circuit. To understand what happens next, it's perhaps easier to consider that the switch in the figure above has a very small (but finite) resistance when closed.

Current will flow from the voltage source to the inductor through the switch provided the voltage source V_S is at a higher potential than the back emf generated by the inductor, which is given by:

$$e(t) = L\frac{dI}{dt} \tag{17.5}$$

(note I've removed the minus sign that sometimes put in this equation, since in Figure 17.3 the back emf and the current are shown in opposite directions; see the chapter on electromagnetism for more details about this point).

This defines the rate at which the current increases. If the current tries to increase too slowly, the back-emf $e(t)$ will be lower than V_S, which increases the potential across the switch, which allows a higher current to flow. If the current is increasing too fast, the back-emf is increased, which reduces the potential difference across the switch, which reduces the current. There is a negative feedback effect going on here that maintains the rate of increase of the current at the value suggested by equation (17.5).

Noting then, that:

$$e(t) = V_S \tag{17.6}$$

and integrating equation (17.5) then gives:

$$I(t) = \frac{V_S}{L}\int_0^t dt = \frac{V_S t}{L} \tag{17.7}$$

and again this suggests a current which increases linearly with time, with a gradient of V_S / L.

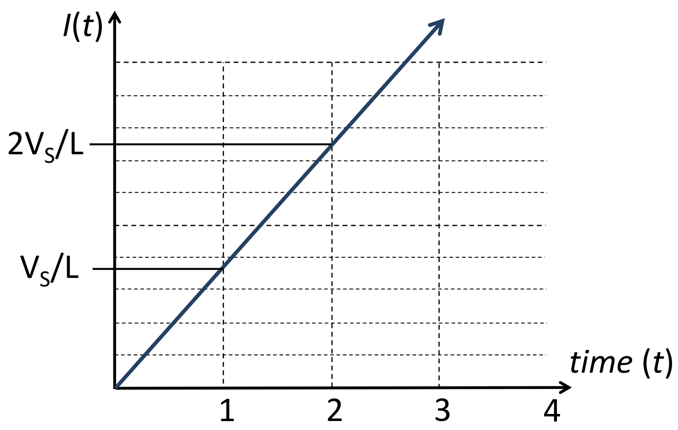

Figure 17.4 Current increasing linearly through an inductor charged from a voltage source

17.4 Capacitors and voltage sources; inductors and current sources

At this point you might be wondering what happens if a capacitor is connected across a voltage source, or an inductor is connected across a current source.

Neither can happen. In the former case, if the capacitor was connected directly across a constant voltage source, then in theory the charge on the capacitor would have to instantly change from zero to $Q = CV$. An instantaneous change in charge implies an infinite current. No real voltage source can supply an infinite current.

Equally, with an inductor connected to a constant current source, then in theory the current through the inductor would have to instantaneously change from zero to the value of the current source, and that would generate an infinite back-emf. No real inductor can generate an infinite back-emf.

(What would happen in these circumstances in the lab is that the non-ideal characteristics of the components would become important, limiting the maximum currents and voltages in the circuit. For example, connect a battery to a capacitor, and the current is limited by the Thévenin equivalent resistance of the battery and the resistance of the wires.)

17.5 Capacitors and op-amps: differentiator and integrators

At this point we can analyse a couple of rather interesting circuits. Firstly, consider a circuit that I'll call an integrator (see Figure 17.5).

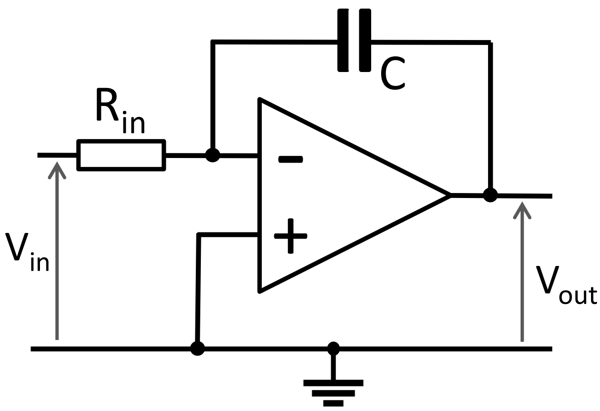

Figure 17.5 An integrator

It's an op-amp configured as an inverting amplifier, but instead of a feedback resistor, there's a capacitor between the output and the inverting input of the op-amp. It can be analysed in the same way as an inverting amplifier: consider that for linear operation of an ideal op-amp the inverting and non-inverting inputs must be at the same voltage, and since the non-inverting input is tied to ground, the inverting input must be at ground potential as well.

So the current flowing in from the input will be V_{in} / R_{in}. However this current now flows onto one plate of the capacitor, depositing charge there. The voltage across the capacitor therefore increases, at a rate of:

$$\frac{dQ}{dt} = i(t) = \frac{V_{in}(t)}{R_{in}} \tag{17.8}$$

Since the voltage across the plates of the capacitor is related to the charge on each plate by $Q = CV$, the voltage across the capacitor must be changing at the rate:

$$\frac{dV_C}{dt} = \frac{1}{C}\frac{dQ}{dt} i(t) = \frac{V_{in}(t)}{R_{in}C} \tag{17.9}$$

and since the left-hand plate of the capacitor is tied to ground (zero volts), the other plate (connected to the output of the op-amp) must now be at $-V_C$ volts. So we have:

$$\frac{dV_{out}}{dt} = \frac{-V_{in}(t)}{R_{in}C} \tag{17.10}$$

Or integrating this up:

$$V_{out}(t) = \frac{-1}{R_{in}C} \int V_{in}(t)\, dt \tag{17.11}$$

Now you can see why this circuit is called an *integrator*: the output is proportional to the integral of the input voltage.

In real life this circuit is rarely used, since if V_{in} does not have an average value of exactly zero, the output of the integrator circuit will eventually saturate. For this reason, it's usual to put a large resistor in parallel with the capacitor (see Figure 17.6), so that very small input voltages don't eventually saturate the op-amp (the small currents they produce through R_{in} can go through R_{leak}, rather than deposit charge on the capacitor). This makes the circuit into a *leaky integrator*.

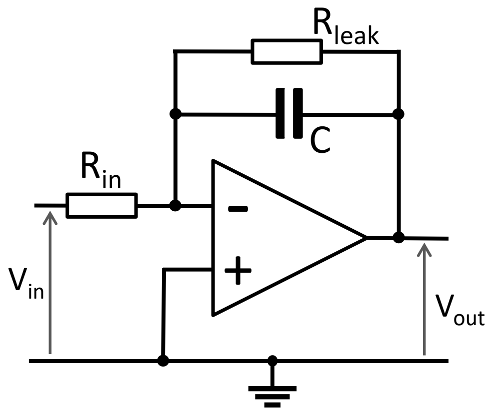

Figure 17.6 A leaky integrator circuit

Next, what about this circuit?

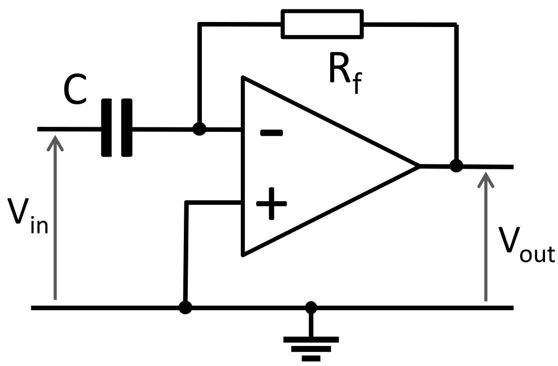

Figure 17.7 A differentiator

You can probably guess what it does from the title of the figure: but why is it doing this?

Here, the voltage across the capacitor is always equal to V_{in} (since the right-hand terminal of the capacitor is held at ground (equal to the non-inverting input voltage) by the action of the op-amp). However, when the voltage across a capacitor changes, the charge held on the plates of the capacitor also changes, and that requires a current to flow. In this circuit, that current flows through R_f, and results in the output of the op-amp taking a non-zero value whenever the potential across the capacitor is changing.

The charge on the capacitor plates is given by:

$$Q = C V_{in}(t) \tag{17.12}$$

so if the voltage input changes, so does the voltage across the capacitor, and hence the current flowing onto one capacitor plate and off from the other (remember that the total amount of charge in a capacitor is constant) is:

$$i(t) = \frac{dQ}{dt} = C\frac{dV_{in}}{dt} \tag{17.13}$$

and this creates a potential difference across the feedback resistor of:

$$i(t)R_f = R_f \frac{dQ}{dt} = R_f C \frac{dV_{in}}{dt} \tag{17.14}$$

Since the left-hand side of the resistor is held at ground (zero volts), and current always flows through a resistor from the higher voltage to the lower voltage, the output of the op-amp must be:

$$V_{out} = -i(t)R_f = -R_f \frac{dQ}{dt} = -R_f C \frac{dV_{in}}{dt} \tag{17.15}$$

and now we have a differentiator: a circuit whose output voltage is proportional to the rate of change of the input voltage.

(You can build similar circuits using inductors rather than capacitors, but since inductors are larger, more expensive and have greater parasitics than capacitors, it's difficult to imagine why you would want to.)

17.6 Discharging a capacitor through a resistor

Back to passive circuits, and the next most interesting case to consider occurs when discharging a capacitor through a resistor. This circuit introduces the concept of a *time constant*.

Consider the circuit shown below, where a capacitor, initially charged up with a voltage of V_0 between its plates, is connected to ground through a switch and a resistor.

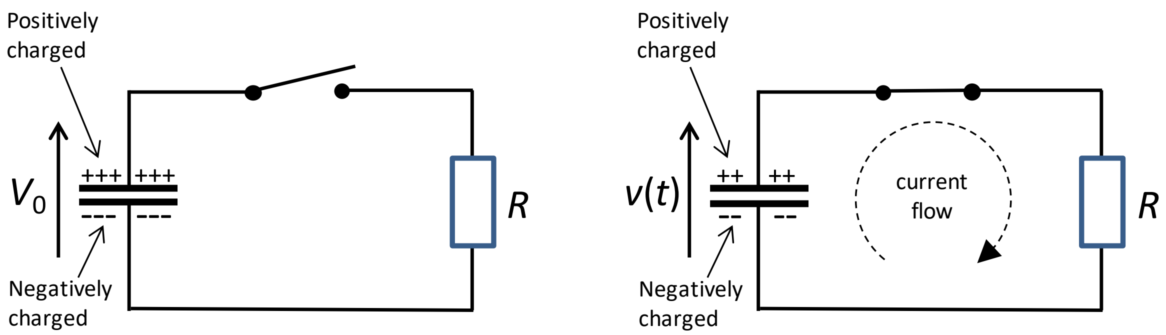

Figure 17.8 A simple R-C circuit for discharging a capacitor at time t = 0 (left) and time t (right)

As soon as the switch is closed, current will start to flow through the resistor (since there is now a potential difference across it). This current will carry a charge that leaves the top-plate of the capacitor, reducing the voltage across the capacitor. This will reduce the potential difference across the resistor, and this will reduce the current flowing, and so on.

Immediately after the switch is closed, the current flowing is V_0 / R (since the voltage across the resistor is just V_0 at the start), but over time this current decreases. After a very long time[1], when the capacitor has fully discharged, there will be no voltage across the resistor, and no current will be flowing at all.

[1] In theory, we have to wait an infinite time for the current to stop completely.

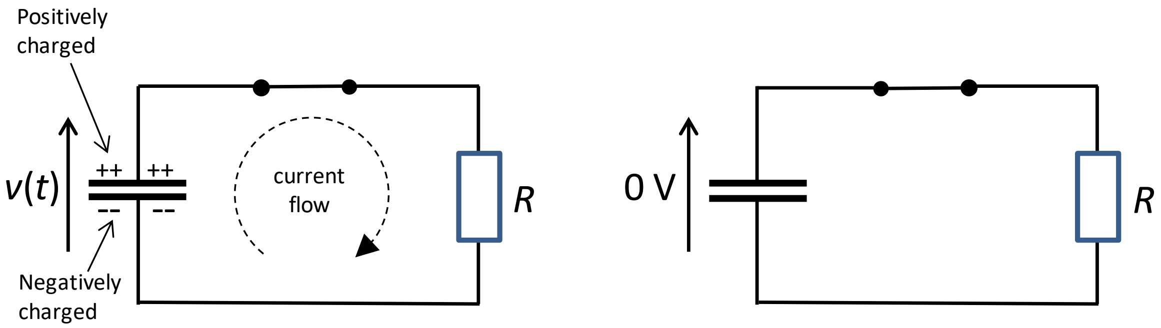

Figure 17.9 Voltages in the circuit at time t (left) and after an infinite time (right)

It would be useful to derive an equation that can tell us the current flowing and the voltage across the terminals of the capacitor at any time. This can be done quite simply as follows: firstly, consider the very small amount of time from t to $t + \delta t$. Let the voltage across the capacitor as a function of time be $v(t)$. Then we can use Ohm's law to determine the current flowing at this time t:

$$i(t) = \frac{v(t)}{R} \tag{17.16}$$

Since the amount of time is very small, we can assume that the current flowing during this time is constant (not enough charge leaves the capacitor to make the voltage across it significantly different). Since charge = current x time, we can work out the total charge transferred off the capacitor's top plate during this short time. I'll call this small amount of charge δq:

$$\delta q = \left(\frac{v(t)}{R}\right)\delta t \tag{17.17}$$

Now since the voltage across a capacitor is related to the charge stored on the capacitor by:

$$Q = CV \qquad\qquad V = \frac{Q}{C} \tag{17.18}$$

the change in the voltage on the capacitor that this reduction in charge will result in, is just:

$$\delta v = \frac{-\delta q}{C} = \left(\frac{-v(t)}{RC}\right)\delta t \tag{17.19}$$

(note the minus sign: a charge leaving will reduce the voltage on the capacitor, which is a negative difference in the voltage).

We know the voltage at time $t = 0$ is V_0, so integrating this between the initial conditions and a time t gives:

$$\begin{aligned}\int_{V_0}^{v(t)} \frac{\delta v}{v(t)} &= -\int_0^t \frac{\delta t}{RC} \\ \ln(v(t)) - \ln(V_0) &= \frac{-t}{RC} \\ \ln\left(\frac{v(t)}{V_0}\right) &= \frac{-t}{RC} \\ v(t) &= V_0 \exp\left(\frac{-t}{RC}\right)\end{aligned} \tag{17.20}$$

This is known as an exponential decay, and it's very common. It looks like this:

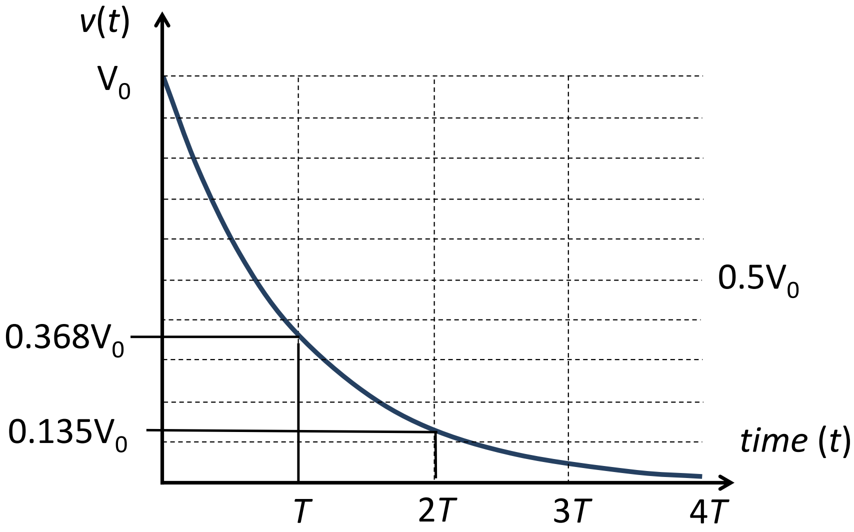

Figure 17.10 Exponential delay of a capacitor discharging

The *time constant* of an exponential decay like this is defined as the time it takes for the voltage to reduce by a factor of 1 / *e* = exp(–1) = 0.36788... This time constant[2] can be readily calculated from this definition, by asking after what time does the voltage reduce by a factor of exp(–1). You might be able to see directly from the equation above that the answer to that question is *t* = *RC*, if not you can arrive at the result more formally: let *T* be the time constant, then:

$$\begin{aligned}v(T) = \frac{V_0}{\exp(1)} &= V_0 \exp(-1) = V_0 \exp\left(\frac{-T}{RC}\right) \\ \therefore\ -1 &= \frac{-T}{RC} \\ \therefore\ T &= RC\end{aligned} \tag{17.21}$$

[2] In general, a time constant is any time that can be used to characterise any behaviour of a system, however in many cases of interest to electronic engineers, the term *time constant of a circuit* has a specific meaning, which indicates how fast the voltages and currents in a circuit are changing in exponential decays.

This result is often more usefully remembered as "in one time constant, the voltage on a capacitor drops to 36.8 % of its initial value". Why 36.8 %? Because if the voltage decreases by a factor of *e*, it drops in value by an amount:

$$\frac{v(t)}{v(0)} = \frac{1}{e} = 0.368... \tag{17.22}$$

These are very important results and worth remembering:

- The time constant of a simple combination of a resistor and a capacitor is the product of the resistance and the capacitance
- The time constant of an exponential capacitor discharge is the time taken for the voltage on the capacitor to reduce to 36.8% of the charge that it started with.

17.6.1 Capacitor discharge characteristics

Once we know the time-constant *T*, we can determine the time it takes for the capacitor to lose any given proportion of its charge. For example, suppose we were interested in the time it takes the capacitor to lose 50% of the charge on each plate. Charge is proportional to the voltage across the capacitor, so this is equivalent to determining the time it takes the capacitor to reach half the voltage it started with:

$$\begin{aligned} v(t) &= \frac{V_0}{2} = V_0 \exp\left(\frac{-t}{T}\right) \\ \exp\left(\frac{-t}{T}\right) &= \frac{1}{2} \\ t &= -T \ln\left(\frac{1}{2}\right) = 0.693T \end{aligned} \tag{17.23}$$

and the same technique works for determining the time taken to lose any fixed proportion of the charge.

Note that it's only the proportion of the remaining charge that's important. The capacitor plates loses half of their charge (from 100% charged to 50% charged) in the same time as it then takes to lose half of their remaining charge (going from 50% charged to 25% charged), and in the same time again they lose half of their remaining charge (going from 25% charged to 12.5% charged) and so on.

As a result, the capacitor takes an infinite time to completely discharge.

17.6.2 What about charging the capacitor up?

Good question. Suppose we started with the capacitor completely uncharged, and connected it to a battery through a resistor and a switch (see circuit below). Then we close the switch.

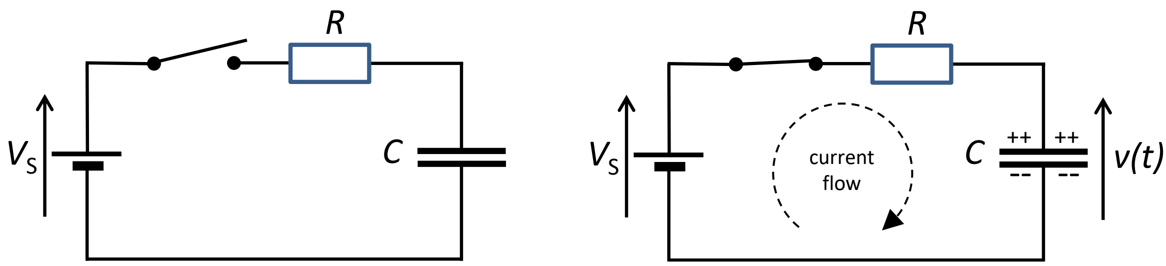

Figure 17.11 A simple R-C circuit for charging a capacitor, just before time $t = 0$ (left), and at time t (right)

In terms of the currents flowing, a very similar thing happens as before: the voltage across the resistor starts off as V_S (V_S is the voltage supplied by the cell and the discharged capacitor has no voltage across it), and so the current starts off being V_S / R. However as the capacitor charges up, the voltage across the resistor reduces, and hence so does the current, and as time goes on, the current flowing gets smaller and smaller.

The analysis is similar, it's just that the current flowing in the resistor is now:

$$i(t) = \frac{V_S - v(t)}{R} \tag{17.24}$$

and in the small time between t and $t + \delta t$ the charge on the capacitor's top plate is now increasing, not decreasing, by:

$$\delta q = i(t)\delta t = \left(\frac{V_S - v(t)}{R}\right)\delta t \tag{17.25}$$

so the change in the voltage on the capacitor that this increase in charge will result in, is:

$$\delta v = \frac{\delta q}{C} = \left(\frac{V_S - v(t)}{RC}\right)\delta t \tag{17.26}$$

The integration limits change as well, since now at time $t = 0$, the voltage across the capacitor is zero:

$$\begin{aligned} \int_0^{v(t)} \frac{\delta v}{V_S - v(t)} &= \int_0^t \frac{\delta t}{RC} \\ -\ln\left(V_S - v(t)\right) + \ln\left(V_0\right) &= \frac{t}{RC} \\ \ln\left(\frac{V_S - v(t)}{V_S}\right) &= \frac{-t}{RC} \\ V_S - v(t) &= V_S \exp\left(\frac{-t}{RC}\right) \\ v(t) &= V_S\left(1 - \exp\left(\frac{-t}{RC}\right)\right) \end{aligned} \tag{17.27}$$

In other words, after one time constant ($T = RC$) the voltage on the capacitor has increased to:

$$v(T) = V_S\left(1 - \exp\left(\frac{-T}{T}\right)\right) = V_S\left(1 - \exp(-1)\right) = 0.632V_S \qquad (17.28)$$

so it gets to about 63.2% of the way to V_S, with only 36.8% of the way to go. After two time constants, the voltage is:

$$v(2T) = V_S\left(1 - \exp\left(\frac{-2T}{T}\right)\right) = V_S\left(1 - \exp(-2)\right) = 0.865V_S \qquad (17.29)$$

about 86.5% of the way to V_S with 13.5% of the way left to go. Note that 13.5% is a factor of 1/e smaller than 36.8%, and this pattern repeats, with the gap between the voltage on the capacitor and the voltage supplied by the battery reducing by a factor of 1/e = 0.368 every time constant.

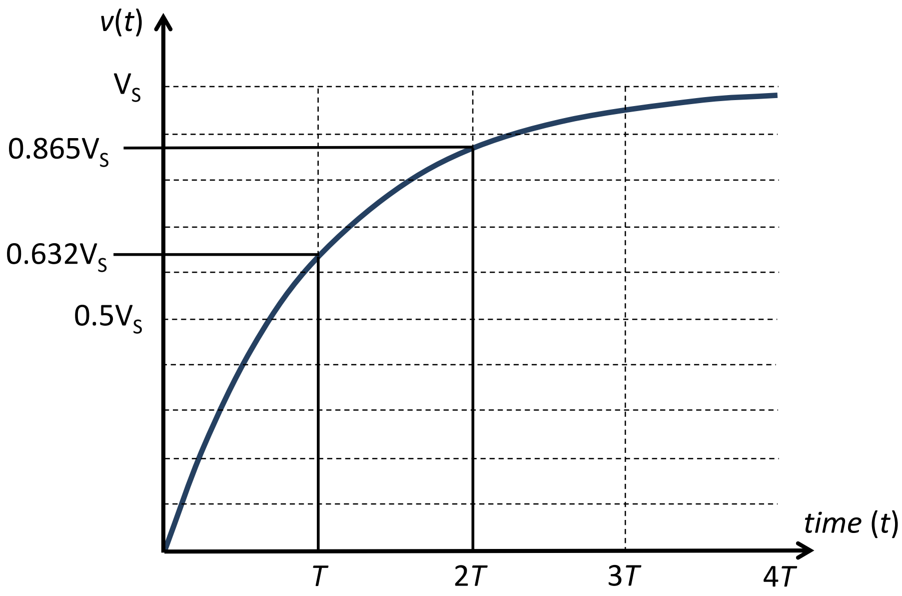

Figure 17.12 The voltage across a capacitor when being charged from a fixed voltage source

17.7 What happens if the voltage source is a sine wave?

Now things get really interesting. Suppose we have a circuit like that shown below, in which the RC (resistor-capacitor) combination is connected in the form of a potential divider, with the input coming from a sine-wave generator. What does the voltage on the capacitor look like now?

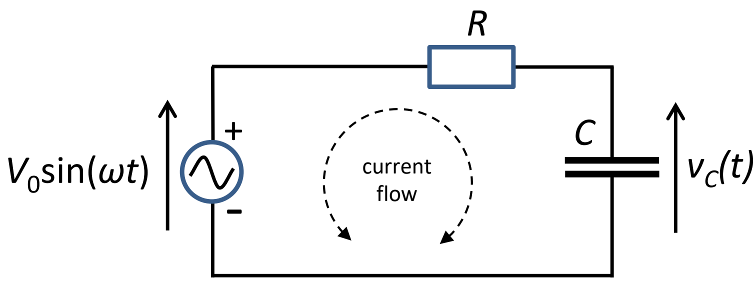

Figure 17.13 Charging and discharging a capacitor from a sinusoidal voltage source

Define the voltage from the sine-wave generator as:

$$V_S(t) = V_0 \sin(\omega t) \tag{17.30}$$

Then if the voltage between the plates of the capacitor is $v_C(t)$, the current flowing through the resistor onto the capacitor will now be:

$$i(t) = \frac{V_0 \sin(\omega t) - v_C(t)}{R} \tag{17.31}$$

so in the small time between t and $t + \delta t$ the charge on the capacitor's top plate is now increasing by:

$$\delta q = i(t)\delta t = \left(\frac{V_0 \sin(\omega t) - v_C(t)}{R} \right) \delta t \tag{17.32}$$

and the change in the voltage on the capacitor that this reduction in charge will result in, is just:

$$\delta v_C = \frac{\delta q}{C} = \left(\frac{V_0 \sin(\omega t) - v_C(t)}{RC} \right) \delta t \tag{17.33}$$

$$\frac{dv_C}{dt} = \left(\frac{V_0 \sin(\omega t) - v_C(t)}{RC} \right) \tag{17.34}$$

After some rather tedious calculus and algebra[3], it turns out that the solution is:

$$v_C(t) = \frac{V_0}{\sqrt{1 + \omega^2 R^2 C^2}} \sin\left(\omega t - \tan^{-1}(\omega RC)\right) \tag{17.35}$$

There are several interesting observations to make about this formula. Firstly, when the angular frequency ω is very small (small enough that ωRC is much less than one, in other words that the frequency is much less than $1/T$ where T is the time constant), the formula can be well approximated by:

$$v_C(t) = V_0 \sin(\omega t) \tag{17.36}$$

in other words the voltage on the capacitor is following the input exactly. This is reasonable: if the input sinusoid is only changing very slowly, then there is plenty of time for the charge to flow across the resistor and charge the capacitor up (or down) to approximately the same voltage as the sinusoidal voltage source before the voltage of the voltage source changes very much.

---

[3] There's no need to do this tedious calculus and algebra, since when you learn about phasors you'll find out about a much easier way to arrive at this result that doesn't involve solving any differential equations.

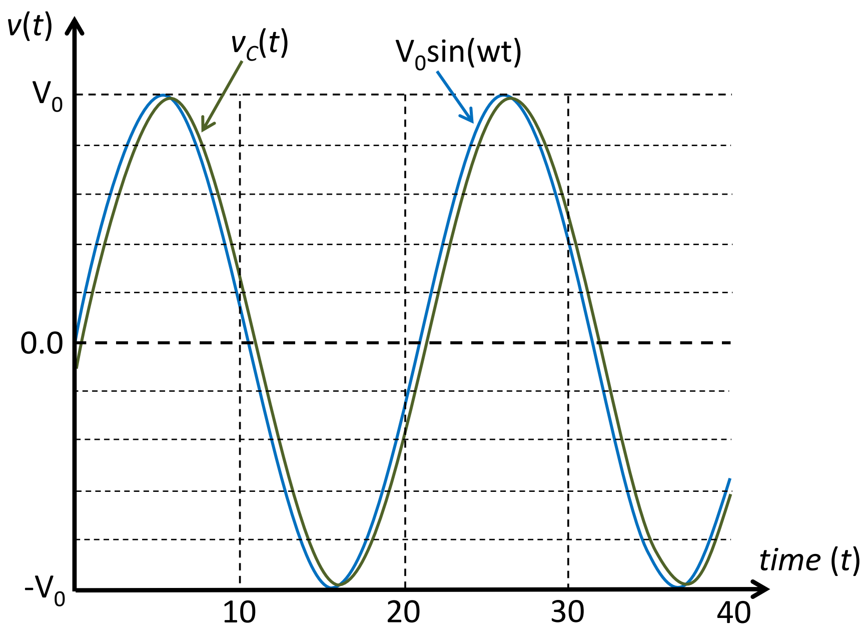

Figure 17.14 Voltage source and capacitor voltage waveforms at a frequency much less than 1/T

On the other hand, when ω is so large that ωRC is much greater than one (in other words the frequency is much greater than $1/T$ where T is the time constant), this formula can be well approximated by:

$$v_c(t) = \frac{V_0}{\omega RC} \sin\left(\omega t - \frac{\pi}{2}\right) \qquad (17.37)$$

This suggest that the phase of the voltage on the capacitor is π/2 radians (90 degrees) out of phase with the input voltage: when the input voltage is at a maximum or minimum, the voltage on the capacitor is zero, and vice versa:

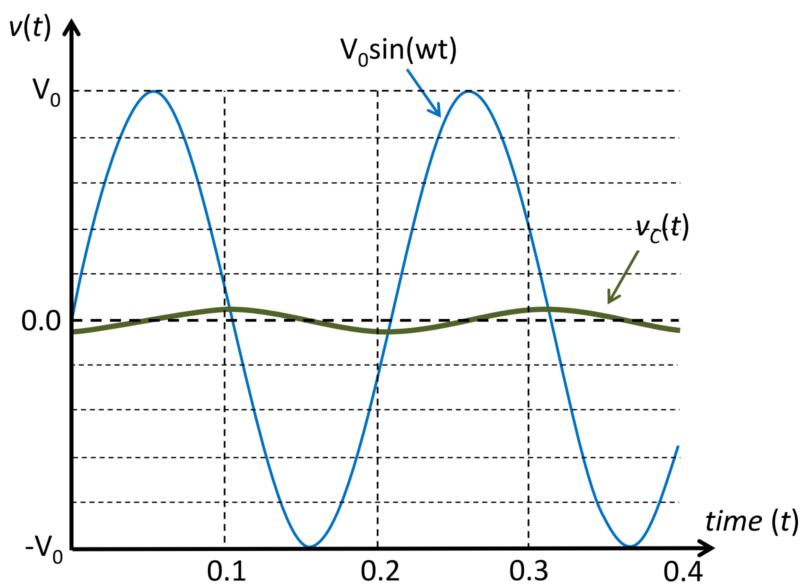

Figure 17.15 Voltage source and capacitor voltage waveforms at a frequency much greater than 1/T

This makes sense if you think about it as well. In this case the input sinusoid is changing in voltage much faster than charge can flow onto, or off from, the capacitor. So the capacitor never has a chance to charge up (or discharge) very much: almost as soon as any charge starts arriving, the voltage from the sinusoidal input will have changed, and start taking charge off again.

Therefore, the voltage on the plates remains close to zero all the time.

During those short times when the sinusoidal voltage is positive, the capacitor will be charging up (slightly), and it will continue to increase in voltage while the input sinusoid is positive (since the input voltage level is greater than the voltage on the capacitor (almost) the whole time the input it positive).

On the other hand, when the when the sinusoidal voltage is negative, the capacitor will be discharging, and it will continue to decrease in voltage while the input sinusoid is negative, since again input voltage level will be more negative than the voltage on the capacitor (almost) the whole time the input is negative.

As a result, the capacitor is continually charging and the voltage across it getting larger all the time the input sinusoid is positive, and continually discharging all the time the input sinusoid is negative. Since we know the voltage on the capacitor is a (much smaller) sinusoid, this suggests that the phase difference between the input and the capacitor voltage will indeed be around 90 degrees (see Figure 17.1 above).

In-between these two extremes, the amplitude of the sinusoidal voltage on the plates of the capacitor moves smoothly between being equal to the input to almost nothing, while the phase difference between the two waveforms increases from zero to around 90 degrees.

Looking at equation (17.35) you might note that there is one other frequency where the equation can be simplified easily, and that's when $\omega = 1/RC$ so that $\omega RC = 1$. At this (angular) frequency, the equation becomes:

$$v_C(t) = \frac{V_0}{\sqrt{2}} \sin\left(\omega t - \frac{\pi}{4}\right) \qquad (17.38)$$

The amplitude has reduced by a factor of $\sqrt{2}$, and the phase difference between the input and the voltage on the capacitor is $\pi/4$ or 45 degrees: exactly half-way between the low-frequency approximation (zero degrees) and the high-frequency approximation (90 degrees).

This frequency is called the *break frequency* and it's related to the time constant by:

$$\text{break frequency (rad/s)} = \frac{1}{\text{time constant}} \qquad (17.39)$$

There's something very interesting (and not at all obvious) about the break frequency, which becomes apparent when you look at the voltage across the resistor. This can be determined quite easily, since knowing the voltage across the capacitor, we can work out the current in the circuit using $i(t) = C\,dV/dt$, and hence the voltage across the resistor using Ohm's law:

$$v_C(t) = \frac{V_0}{\sqrt{1+\omega^2R^2C^2}}\sin\left(\omega t - \tan^{-1}(\omega RC)\right)$$
$$\frac{dv_C(t)}{dt} = \frac{\omega V_0}{\sqrt{1+\omega^2R^2C^2}}\cos\left(\omega t - \tan^{-1}(\omega RC)\right) \tag{17.40}$$

$$v_R(t) = i(t)R = C\frac{dv_C(t)}{dt}R = \frac{RC\omega V_0}{\sqrt{1+\omega^2R^2C^2}}\cos\left(\omega t - \tan^{-1}(\omega RC)\right) \tag{17.41}$$

and when $\omega RC = 1$, this becomes:

$$v_R(t) = \frac{V_0}{\sqrt{2}}\cos\left(\omega t - \frac{\pi}{2}\right) \tag{17.42}$$

so the voltage across the resistor has a magnitude of $V_0/\sqrt{2}$ as well: exactly the same the capacitor.

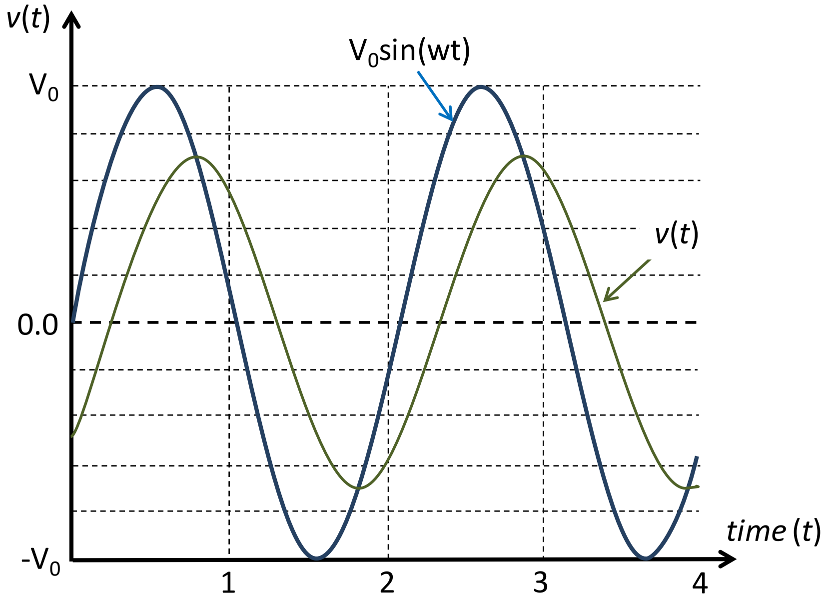

Figure 17.16 Source and capacitor voltages when the angular frequency is the inverse of the time period

17.7.1 Hang on a minute…

At the break frequency, the voltage across the resistor is $V_C/\sqrt{2}$, and the voltage across the capacitor is also $V_C/\sqrt{2}$? But $V_C/\sqrt{2} + V_C/\sqrt{2} = \sqrt{2}\,V_C$ and that's greater than V_C, the voltage from the voltage source. Doesn't that contradict Kirchhoff's voltage law?

No, it doesn't. The point is that it's only the magnitude of the voltage across the capacitor and the voltage across the resistor that are equal. They're not equal all the time, and they don't reach their maximum values at the same time. If you plot the three voltages on an oscilloscope, you get something like Figure 17.17.

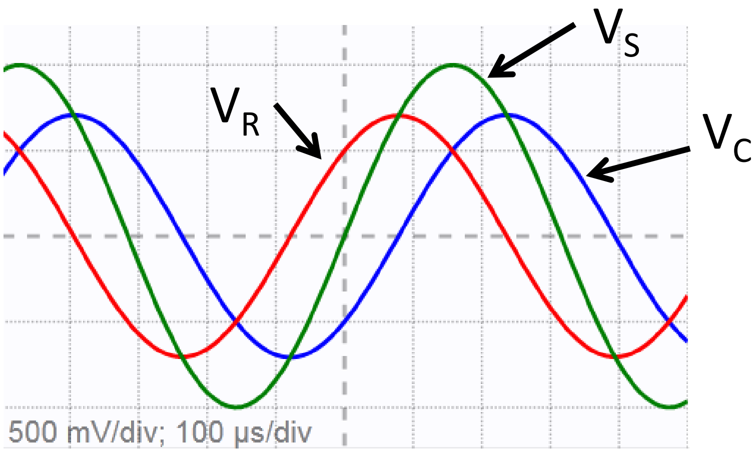

Figure 17.17 The voltages in an RC circuit at the break frequency

Notice that the voltage across the resistor (V_R), and the voltage across the capacitor (V_C), are out-of-phase (they don't reach their maximum values at the same time), and that at every moment in time the voltage across the voltage source (V_S) is equal to the sum of the voltage across the capacitor and the voltage across the resistor. So Kirchhoff's voltage law is always obeyed.

17.8 Summary: the most important things to know

- Capacitors charge linearly from a current source, and exponentially through a resistor from a voltage source, with a time constant of $T = RC$.
- Inductors charge linearly from a voltage source, and exponentially though a resistor from a current source with a time constant of $T = L / R$. (I haven't derived this one, I'll leave it as an exercise for the interested reader.)
- It is impossible to change the voltage across an ideal capacitor instantly.
- It is impossible to change the current through an ideal inductor instantly.
- At the *break frequency* of an RC circuit with a sinusoidal input:
    - the voltage across the capacitance and the voltage across the resistance have the same magnitude
    - the magnitude of the voltage across the capacitor and resistor are equal to the voltage from the voltage source divided by $\sqrt{2}$