15 A Short Introduction to Feedback

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Prerequisite knowledge required: Op-Amps, Thévenin's Theorem

15.1 Introduction

Feedback is the process of taking a portion of the output of a circuit and subtracting it from (or occasionally adding it to) the input of the same circuit. Subtracting some of the output from the input, thus making the input smaller and reducing the gain of the amplifier, might seem like a rather odd thing to do at first sight[1]. However, as we'll see, there are a lot of advantages in doing this, and in particular this principle allows the design of very high-performance circuits using less-than-ideal components.

On the other hand, adding some of the output to the input, thus increasing the gain, is perhaps a more obvious thing to do since it seems to allow the manufacture of very-high gain amplifiers very simply. However, this has far fewer applications in most circuits and can be dangerous; more about this later as well.

15.2 Feedback: the basic idea

A basic block diagram of an amplifier with some feedback is shown below:

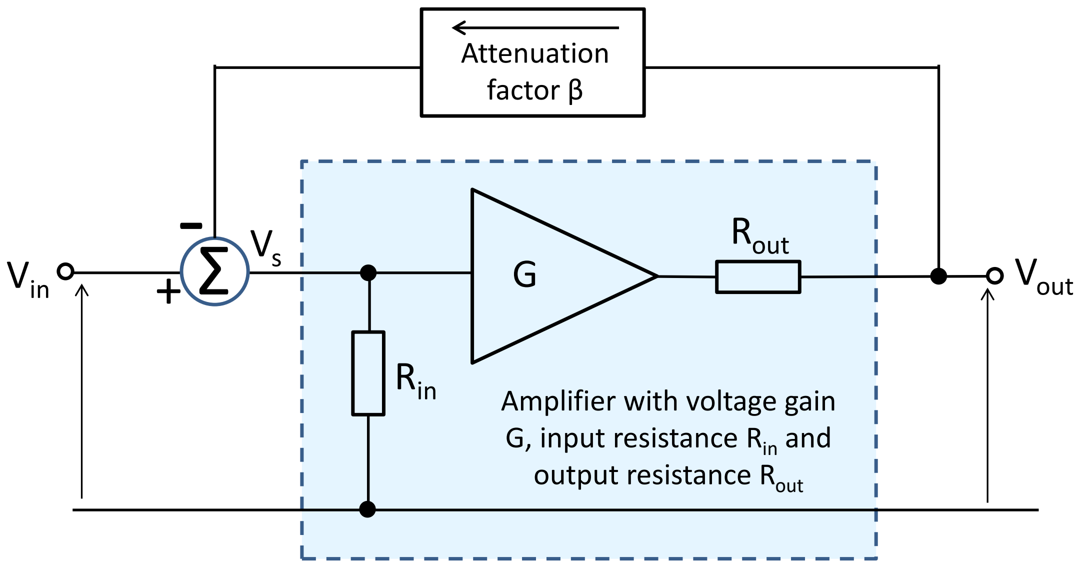

Figure 15.1 Block diagram of a negative feedback amplifier

The diagram might need a bit of explanation. The original amplifier (shaded) is shown as the combination of its input resistance R_{in}, output resistance R_{out}, and a perfect voltage amplifier with a voltage gain of G. (This perfect voltage amplifier has an infinite input resistance and a zero output resistance, a bit like an ideal op-amp.) The block marked "Attenuation factor β" takes the output of the amplifier and multiplies it by an attenuation factor β (it's called an *attenuation factor* since β is less than one). The circle with the capital Greek sigma in it is a summing junction: it takes +1 times

[1] In fact, the inventor of the negative feedback amplifier (Harold Black) was initially refused a patent on the grounds that that patent office thought it was a silly idea and didn't believe it would work.

the input from V_{in} and -1 times the input from the attenuator block, and outputs the result (shown here as V_s); this result is the actual input to the amplifier block. Mathematically, we could write:

$$V_{out} = GV_s = G\left(V_{in} - \beta V_{out}\right) \tag{15.1}$$

Some algebra on this results in the equation:

$$\begin{aligned} V_{out}\left(1+\beta G\right) &= GV_{in} \\ \frac{V_{out}}{V_{in}} &= \frac{G}{\left(1+\beta G\right)} \end{aligned} \tag{15.2}$$

From which it can be readily seen that the voltage gain of the amplifier with feedback is smaller than the voltage gain of the voltage gain of the amplifier without feedback by a factor of $(1 + \beta G)$.

15.3 The advantages of negative feedback

If all you've done is reduce the gain of an amplifier, you haven't achieved much of interest. After all, if the gain of the amplifier you have is too large for your application, it would be much easier to just put a potential divider on the input to lower the input level. Why go to all the bother of building a summing junction? In fact, there are a few good reasons.

First, notice what happens when the gain of the original amplifier is very large. So large, that βG (known as the loop gain[2]) is much greater than one, and we can approximate the denominator as:

$$1+\beta G \approx \beta G \tag{15.3}$$

Then the gain becomes:

$$\frac{V_{out}}{V_{in}} = \frac{G}{\left(1+\beta G\right)} \approx \frac{G}{\beta G} = \frac{1}{\beta} \tag{15.4}$$

In other words, the gain of the feedback amplifier is a function of the attenuation factor β, but is independent of the gain of the original amplifier.

This is very useful. It is difficult to make a high gain amplifier with a good frequency response (in other words one that has the same amount of gain at all frequencies), and yet to give very accurate amplification over a wide range of frequencies this is exactly what is required. However, it's very easy to make attenuators that have the same attenuation for all frequencies of interest: all you need is a couple of resistors wired up as a potential divider. Putting negative feedback around an amplifier can result in much less distortion and better frequency response, since with a very large loop gain the gain is then determined by the feedback resistors, not by the original amplifier itself.

[2] βG, the product of the gain of the op-amp and the "gain" of the attenuation block is known as the loop gain since it is the total gain around the feedback loop (the amplifier and the feedback path). (I've put the second "gain" in that sentence in quotes because it's less than one, so although technically called a gain (because it's the output divided by the input) it actually results in a smaller signal.)

Even if the gain of the amplifier is not so large that βG can be assumed to be much greater than one, employing negative feedback in this way can still improve the frequency response of the amplifier.

This is possibly the main reason negative feedback amplifiers are so popular, but there are other reasons as well. Depending on how the summing block and feedback blocks are implemented in the circuit, feedback can also increase the input impedance and decrease the output impedance of amplifiers using feedback, and thus make circuit loading effects less serious.

In the next sections, we'll examine this effect for a non-inverting amplifier.

15.4 Negative feedback and input impedances

In order to illustrate the effect, I'll have to use an amplifier that behaves as if it has a resistor connected between two inputs: an inverting and a non-inverting input, as shown in Figure 15.2. This is a reasonable model to use for small changes in input signals to an op-amp (note this analysis works for AC signals only, it doesn't work for the total DC input currents[3]).

Treating only the small changes in signals is a common technique in circuit analysis (known as a small-signal model, see the chapter on "Linearity and Superposition" for more details), so for this analysis I'll assume that the amplifier does behave as if it has a resistor between its two inputs, and can be modelled as shown in Figure 15.2. Note that this differential input resistance appears *between* the two inputs. This figure shows a standard non-inverting amplifier[4], and here I've considered the input impedance only and set the output impedance to zero to keep the maths a bit easier.

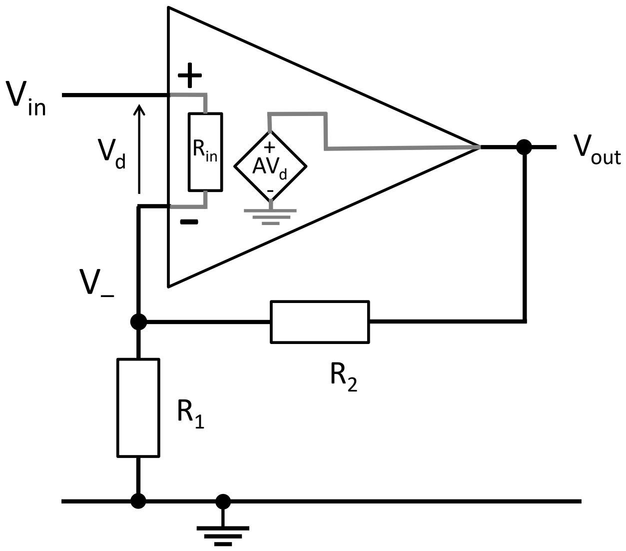

Figure 15.2 Simplified non-inverting amplifier showing input impedance

[3] DC current flows into the input terminals of an op-amp even when $V_+ = V_-$, and these *bias currents* are specified on the op-amp data sheets. The "input impedance" sometimes seen on op-amp datasheets is a differential AC input impedance as shown in the figure above. See the chapter on "Non-Ideal Op-Amps" for more details.

[4] This is the more interesting case. The input impedance of an inverting amplifier is dominated by the resistance between the amplifier circuit input and the inverting input of the op-amp (which is a virtual ground provided the gain of the op-amp is large), and is not a strong function of the feedback.

While the op-amp's differential input impedance (shown as R_{in} in the figure above) is often very large already, the effect of the negative feedback is to increase it still further. There is an intuitive method to understand why this happens: an ideal op-amp in its linear region of operation acts to make the voltages at the inverting and non-inverting inputs equal. If the voltages at the two inputs are equal, and therefore the voltage across the input resistor R_{in} is zero, then no current will flow in R_{in}, and if no current is flowing in R_{in}, the effective input resistance is infinite.

To determine the actual effective input impedance for a non-ideal op-amp, some circuit analysis is required.

Firstly, the output voltage from the amplifier is a factor *A* times the difference in the input voltages at the two terminals, therefore:

$$V_{out} = A\left(V_{in} - V_{-}\right) \tag{15.5}$$

and since the current through the feedback resistor R_2 plus the current through the input resistor R_{in} must be the current through resistor R_1, applying Kirchhoff's current law to the circuit node at the inverting input of the op-amp gives:

$$\frac{A\left(V_{in} - V_{-}\right) - V_{-}}{R_2} + i_{in} = \frac{V_{-}}{R_1} \tag{15.6}$$

where i_{in} is the input current, which from Ohm's law across the input resistance must also be equal to:

$$i_{in} = \frac{V_{in} - V_{-}}{R_{in}} \tag{15.7}$$

The effective input impedance (the input voltage divided by the input current) then given by:

$$R_{eff} = \frac{V_{in}}{i_{in}} \tag{15.8}$$

Eliminating i_{in} and V_{-} between these three equations (which takes rather a lot of tedious algebra) the effective input resistance turns out to be:

$$R_{eff} = \frac{R_{in}}{1 - \dfrac{A + \dfrac{R_2}{R_{in}}}{A + 1 + \dfrac{R_2}{R_{in}} + \dfrac{R_2}{R_1}}} \tag{15.9}$$

Assuming the input impedance is much larger than the resistors in the circuit (the usual case), we can neglect the terms in R_2 / R_{in}, and noting that the ideal closed-loop gain of this amplifier is given by:

$$G = 1 + \frac{R_2}{R_1} \tag{15.10}$$

we can approximate the effective input impedance as:

$$R_{eff} \approx \frac{R_{in}}{1 - \frac{A}{A+G}} \tag{15.11}$$

This reveals that the effective input impedance is always larger than the value of input resistor from the op-amp datasheet, and as long as the closed-loop gain of the amplifier G is much less than the open-loop gain of the op-amp A so that $A + G \approx A$, the effective input impedance of the circuit is enormous.

15.5 Negative feedback and output impedance

The negative feedback typically used in these circuits has an effect on the output impedance as well. This time we'll use a standard model for the op-amp (with an infinite input impedance), except we'll include the open-loop output impedance (this is often specified in the data sheet, and is typically around 100 ohms), see Figure 15.3.

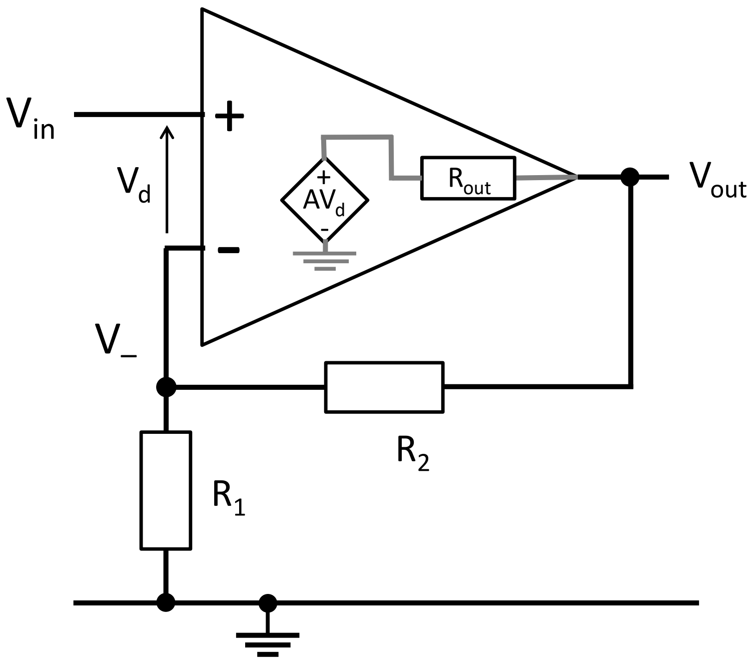

Figure 15.3 Simplifier non-inverting amplifier showing output impedance

The output is modelled as a Thévenin equivalent circuit with an open-circuit voltage equal to the gain A times the difference in the input voltages ($V_d = V_{in} – V_–$), and a Thévenin-equivalent resistance of R_{out}.

The effective output impedance of the amplifier circuit can be obtained by calculating the output voltage with no load attached (the open-circuit output voltage) and the output current when the output is shorted to ground (the short-circuit output current). The ratio of these gives the effective output impedance[5].

First, the open-circuit output voltage. Since the output of the op-amp is being modelled as a Thévenin-equivalent voltage source of value:

$$V_{internal} = A\left(V_{in} - V_{-}\right) \tag{15.12}$$

driving out through a Thévenin-equivalent resistance of R_{out}, the actual voltage present at the output of the op-amp circuit can be determined from the standard potential-divider equation:

$$V_{out} = V_{internal}\frac{R_1 + R_2}{R_1 + R_2 + R_{out}} \tag{15.13}$$

where $V_{internal}$ is the voltage of the dependent source inside the op-amp, and similarly the voltage at the inverting input can be given by:

$$V_{-} = V_{internal}\frac{R_2}{R_1 + R_2 + R_{out}} \tag{15.14}$$

Eliminating $V_{internal}$ and V_{-} between the three equations above gives:

$$V_{out} = \frac{AV_{in}\left(R_1 + R_2\right)}{R_1 + \left(A+1\right)R_2 + R_{th}} \tag{15.15}$$

Next, the short-circuit current[6]. Since in this case the output of the op-amp is being held at ground, the voltage at the inverting input must be ground as well (we are neglecting the input impedance for this calculation, so the inverting input is just connected to ground through R_1 and ground through R_2).

This means that the current flowing through the op-amp's output impedance must be:

$$I_{SC} = \frac{A\left(V_{in} - 0\right)}{R_{out}} \tag{15.16}$$

Therefore, the Thévenin equivalent output impedance of this amplifier is:

$$R_{th} = \frac{V_{OC}}{I_{SC}} = \frac{\dfrac{AV_{in}\left(R_1 + R_2\right)}{R_1 + \left(A+1\right)R_2 + R_{th}}}{\dfrac{A\left(V_{in} - 0\right)}{R_{out}}} = \frac{\left(R_1 + R_2\right)R_{out}}{R_1 + \left(A+1\right)R_2 + R_{th}} \tag{15.17}$$

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[5] See the chapter on Thévenin and Norton if unsure of this step.

[6] It's important to note that this is a theoretical calculation only, for the purposes of determining a Thévenin equivalent circuit for the output of this amplifier. Grounding the output of a real op-amp would either result in a much lower current (due to current-limiting circuitry in the op-amp), or damage to the op-amp.

and since $(R_1 + R_2)$ must be less than $(R_1 + (A + 1)R_2 + R_{th})$, the output impedance of the amplifier is always less than the output impedance of the op-amp itself.

When A is very large, the denominator of this expression is dominated by the term in $A R_2$, so we can approximate:

$$R_1 + (A+1)R_2 + R_{th} \approx A R_2 \tag{15.18}$$

and write the effective output impedance as:

$$R_{th} \approx \frac{(R_1 + R_2)R_{out}}{AR_2} \tag{15.19}$$

Further, noting that the gain of the non-inverting amplifier is $G = (R_1 + R_2) / R_2$, we could write:

$$R_{th} \approx \frac{G}{A} R_{out} \tag{15.20}$$

which gives the useful result that the output impedance of the op-amp itself has been reduced by a factor of approximately G/A, the ratio of the closed-loop gain to the open-loop gain.

This can be a very large effect: consider some typical numbers, for example $A = 100{,}000$, $R_{out} = 100$, and $G = 10$. Then the effective output impedance of the amplifier is:

$$R_{th} \approx \frac{10}{100{,}000} \times 100 = 0.01\ \Omega \tag{15.21}$$

At this point, I hope you can see how one of the properties of an ideal op-amp (that it has a zero output impedance) can be closely approximated in real circuits, even if the op-amp itself has a significant equivalent output impedance.

15.6 Positive feedback

What about positive feedback? Suppose you built an amplifier which added a small amount of the output voltage to the input voltage rather than subtracting it. Going through the same calculations as before leads to the equation:

$$\frac{V_{out}}{V_{in}} = \frac{G}{(1 - \beta G)} \tag{15.22}$$

Now if you set β so that it is almost, but not quite, equal to $1/G$, the result is that βG is almost, but not quite, equal to one. This in turn means that $1 - \beta G$ is very small indeed, which makes the gain of the amplifier enormous. In theory, the gain can be made much greater than the gain of the original amplifier, G. This is very useful if you want to increase the gain of your amplifier.

However, there is a danger. If you misjudge this so that βG becomes equal to one, then the gain of the feedback amplifier becomes, at least in theory, infinite. Which isn't possible of course, so what happens for an amplifier with an infinite gain at DC is that the amplifier just picks up any small amount of noise at its input (there is always some noise present in a real circuit), heads off to either positive or negative saturation and stays there.

(It is possible to build an amplifier circuit with feedback which gives this critical amount of positive feedback at some frequency other than DC, but much less feedback at DC. Such circuits are more commonly called *oscillators*, since what they do is produce as much amplitude as they can at a frequency at which they have an infinite gain.)

15.6.1 Comparators

There is however one circumstance in which a circuit with an infinite gain might be exactly what you want. There is a circuit called a *comparator*, which aims to produce an output which has one of only two possible values, depending on whether one input is above or below the other input. You can make a basic comparator using an op-amp, just by leaving off the feedback resistor entirely:

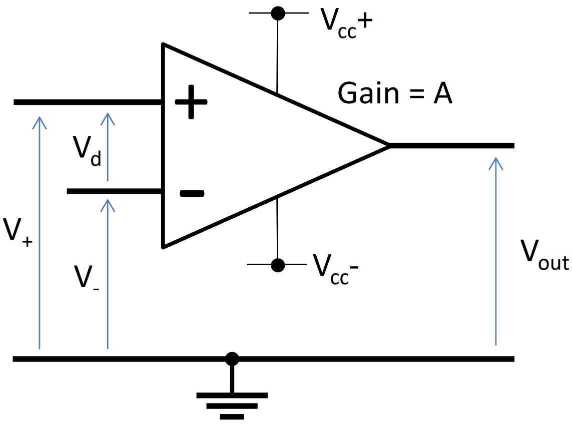

Figure 15.4 Op-Amp as a comparator

In theory, the output of such an op-amp will try to reach:

$$V_{out} = A\,V_d = A\left(V_+ - V_-\right) \tag{15.23}$$

where *A* is typically a very large gain. In fact, it's so large that for any reasonable difference in the two inputs the output will saturate at either the maximum possible positive or maximum possible negative output voltage (the closest it can get to the value predicted by equation (15.23) given the fact that a real op-amp can't output a voltage outside the range of its power supplies).

Adding positive feedback to the comparator circuit above results in a circuit called a Schmitt trigger (see the chapter about non-linear op-amp circuits for more details about this extremely useful circuit).

15.7 Summary: the most important things to know

- Negative feedback allows very linear amplifiers to be produced using amplification circuits which themselves are not linear.
- Negative feedback can also increase the input resistance and decrease the output resistance of circuits.
- Positive feedback can increase the speed of comparators, and make comparators immune to noise in their input signals (see the chapter about non-linear op-amp circuits).