14 A Short Introduction to Linear Operational Amplifier Circuits

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Prerequisite knowledge required: Op-Amps, Ohm and Kirchhoff's Laws, Circuit Loading, Linearity and Superposition

14.1 Introduction

There are hundreds of useful op-amp circuits. All I have room for here is a short discussion of a couple of the most useful ones, and in particular the circuits that are used in the remainder of this module and are not covered elsewhere. This means this chapter will cover:

- The summing amplifier
- The differential amplifier

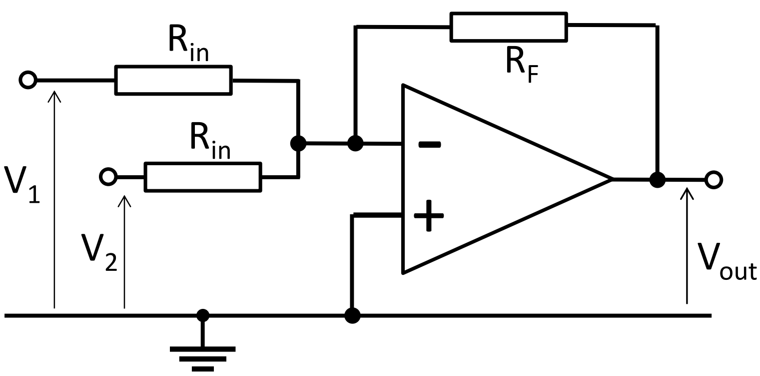
(The inverting amplifier and non-inverting amplifier were covered in the introduction to op-amps; active filters, comparators and Schmitt triggers, multipliers and dividers, log-amps and active rectifiers are covered in the chapters on "Active Filters" and "Non-Linear Op-Amp Circuits".)

Summing amplifiers can output the sum of two (or more) input voltages, differential amplifiers output the difference between two input voltages. Both are very common requirements, for example summing amplifiers in audio mixers, and differential amplifiers in communications systems using twisted pair cables.

14.2 Summing amplifiers

The function of a summing amplifier is to add together two or more signals. Most summing amplifiers built using op-amps are based on inverting amplifiers, so the output voltage is actually minus one times the sum of the input voltages.

A basic summing amplifier with two inputs is shown below:



$$V_{out} = -\frac{R_F}{R_{in}}V_1 - \frac{R_F}{R_{in}}V_2$$

Figure 14.1 An inverting summing amplifier with two inputs

This circuit can be analysed in much the same way as the inverting amplifier described in the introduction to op-amps chapter: first notice that the inverting input to the op-amp must be very close to zero volts (since the non-inverting amplifier input is wired directly to ground), and then consider the currents flowing from each input towards the op-amp's inverting input. This total current is given by:

$$I_{in} = \frac{V_1}{R_{in}} + \frac{V_2}{R_{in}} \tag{14.1}$$

This current then has to flow through the feedback resistor R_F (it has nowhere else to go since no current flows into the inputs of an ideal op-amp), and will therefore generate a voltage across that resistor of:

$$I_{in}R_F = R_F\frac{V_1}{R_{in}} + R_F\frac{V_2}{R_{in}} \tag{14.2}$$

However, the voltage across this resistor can also be written in terms of the output of the op-amp, and the fact that the left-hand side of the resistor is at zero volts. Applying Ohm's law to this feedback resistor give the voltage across the feedback resistor as:

$$V_F = 0 - V_{out} = I_{in}R_F \tag{14.3}$$

Combining equations (14.2) and (14.3) gives:

$$V_{out} = -R_F\frac{V_1}{R_{in}} - R_F\frac{V_2}{R_{in}} = -\frac{R_F}{R_{in}}\left(V_1 + V_2\right) \tag{14.4}$$

In other words, the output is an inverted, amplified version of the sum of the two input voltages.

14.2.1 Superposition

There's another way to work out how the previous circuit works, and that's to use the principle of superposition. This states that:

"For any linear circuit, the output given a number of inputs is equal to the sum of the outputs given by each input in turn."

This is a linear circuit (double all the inputs and the output doubles as well), so we could consider the previous circuit one input at a time, with the other input grounded in each case:

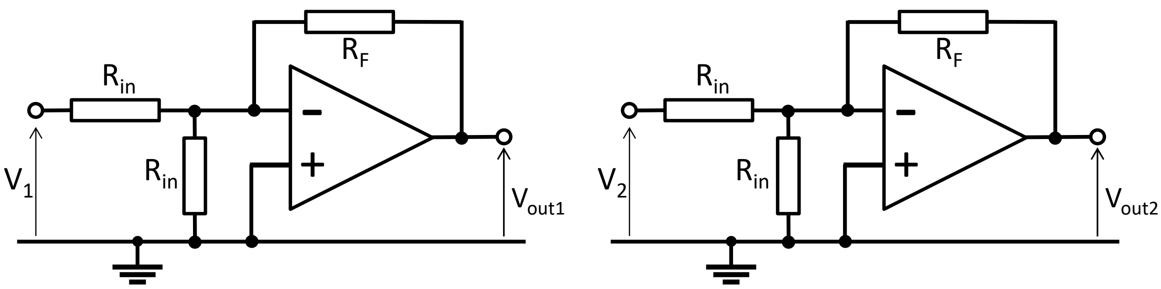


Figure 14.2 Summing amplifier with input 2 grounded (left) and input one grounded (right)

In both cases, this is just an inverting amplifier, with another resistor connected between the inverting input and ground. However, since with an ideal op-amp the inverting input would be at zero volts anyway, there will be no current flowing through this resistor, so we can ignore it.

This means that the output we get from the first amplifier using the standard equation for a single-input inverting amplifier:

$$V_{out1} = -\frac{R_F}{R_{in}} V_1 \tag{14.5}$$

and the output from the second amplifier is similarly:

$$V_{out2} = -\frac{R_F}{R_{in}} V_2 \tag{14.6}$$

So since this is a linear circuit, we can write the output when both inputs are used:

$$V_{out} = V_{out1} + V_{out2} = -\frac{R_F}{R_{in}} V_1 - \frac{R_F}{R_{in}} V_2 = -\frac{R_F}{R_{in}} \left(V_1 + V_2 \right) \tag{14.7}$$

For more complex circuits, this is often a very useful way to calculate the total output given a number of different inputs.

14.2.2 More general summing amplifiers

Of course there's no reason that a summing amplifier has to have only two inputs, and equally there's no reason why a summing amplifier has to have the same gain from each of its inputs to its output. For example, consider the circuit shown below:

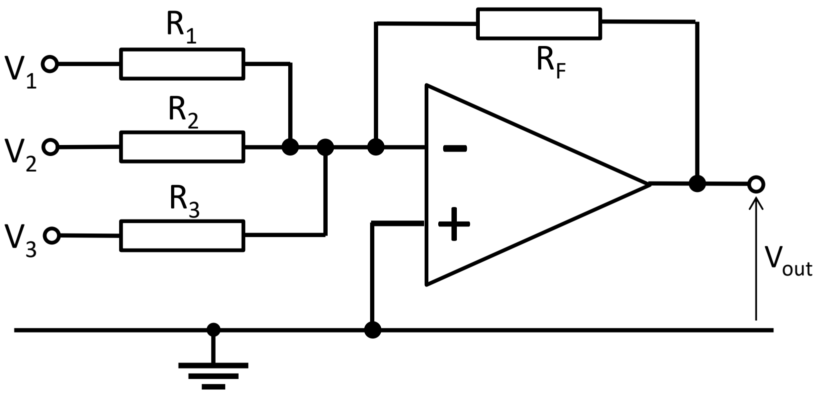


Figure 14.3 Summing amplifier with three inputs

This circuit has three inputs, from which current flows through three different resistors towards the inverting input of the op-amp. Again applying the principle of superposition, we can consider each input in turn, for example for the first input V_1:

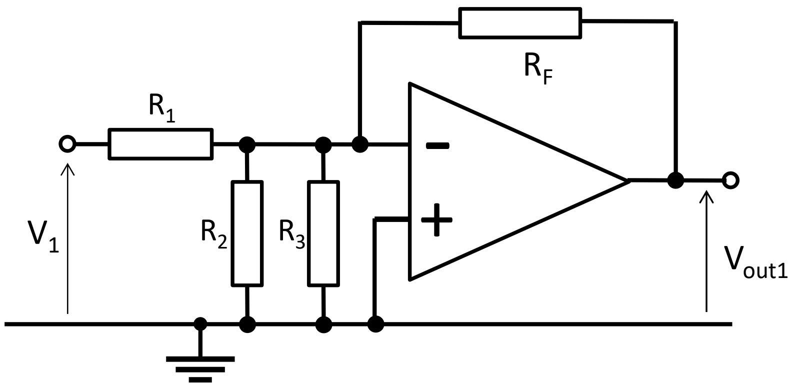


Figure 14.4 Three-input summing amplifier with two inputs grounded

and in each case we just have an inverting amplifier. With only the first two inputs connected (and the other two inputs grounded), this gives an output of:

$$V_{out1} = -\frac{R_F}{R_1}V_1 \tag{14.8}$$

and similarly, connecting just the second, or just the third gives:

$$V_{out2} = -\frac{R_F}{R_2}V_2 \qquad\qquad V_{out3} = -\frac{R_F}{R_3}V_3 \tag{14.9}$$

The total output from the circuit will then be the sum of the outputs from applying each input in turn, which suggests a final output of:

$$V_{out} = V_{out1} + V_{out2} + V_{out3} = -\frac{R_F}{R_1}V_1 - \frac{R_F}{R_2}V_2 - \frac{R_F}{R_3}V_3 = -R_F\left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}\right) \tag{14.10}$$

Note that this amplifier gives a weighted sum of the inputs: the gains from each input are no longer the same. Sometimes that's useful.

14.2.3 Another generalisation: don't ground the non-inverting input

Another variation of the summing amplifier occurs when an offset is required on the result, and this can be easily done by attaching the non-inverting input of the summing amplifier to some reference voltage V_{ref} rather than attaching it to ground. The result is the reference voltage minus the sum of the differences between the inputs and this reference voltage[1].

[1] That sentence might benefit from an example: suppose the reference voltage is 1.5 V, and the two input voltages are 2.0 V and 2.2 V. The sum of the differences between the inputs and the reference voltages are (2.0 – 1.5) + (2.2 – 1.5) = 0.5 + 0.7 = 1.2, and subtracting this from the reference voltage gives a net output of 1.5 – 1.2 = 0.3 V.

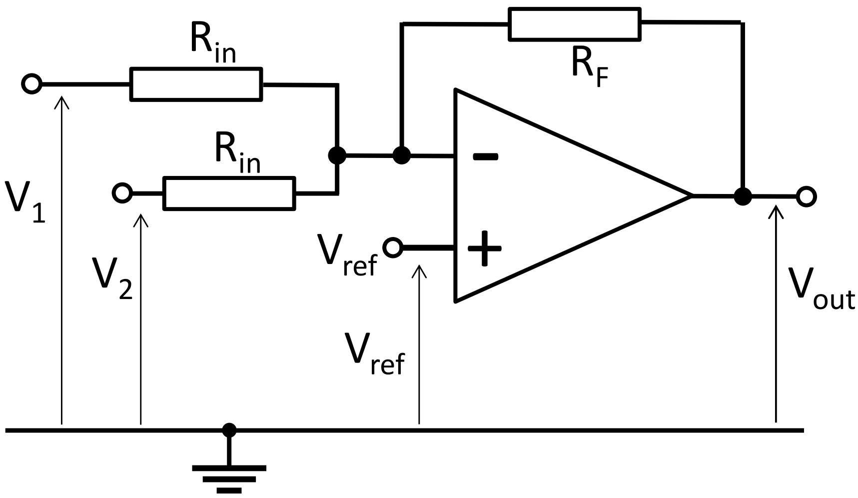


Figure 14.5 Summing amplifier with offset

To see this mathematically, consider the current now flowing in the resistors. The current flowing from the inputs through the input resistors will now be:

$$I_{in} = \frac{V_1 - V_{ref}}{R_{in}} + \frac{V_2 - V_{ref}}{R_{in}} \tag{14.11}$$

and therefore the voltage across the feedback resistor will be:

$$V_f = \left(\frac{V_1 - V_{ref}}{R_{in}} + \frac{V_2 - V_{ref}}{R_{in}} \right) R_f \tag{14.12}$$

Since the inverting input will be at (approximately) the same voltage as the non-inverting input, this makes the output voltage equal to:

$$V_{out} = V_{ref} - V_f = V_{ref} - \left(\frac{V_1 - V_{ref}}{R_{in}} + \frac{V_2 - V_{ref}}{R_{in}} \right) R_f \tag{14.13}$$

Effectively, the circuit is behaving as a summing amplifier with ground raised up to the voltage on the non-inverting input. This circuit is common in the case of single-supply op-amp circuits, where the reference voltage V_{ref} is often half-way between ground and the positive supply[2].

14.2.4 Why put a resistor between the non-inverting input and ground?

Most often, you'll see summing amplifiers (and inverting amplifiers) designed like this:

[2] Particularly alert readers might have noticed that this is not a surprising result at all, since the ground voltage is arbitrary (we are free to define any voltage in the circuit as ground), and if you define the zero of voltage to be the voltage on the non-inverting input, this circuit is exactly the same as the conventional op-amp with a dual-rail supply (being supplied with both positive and negative power supplies).

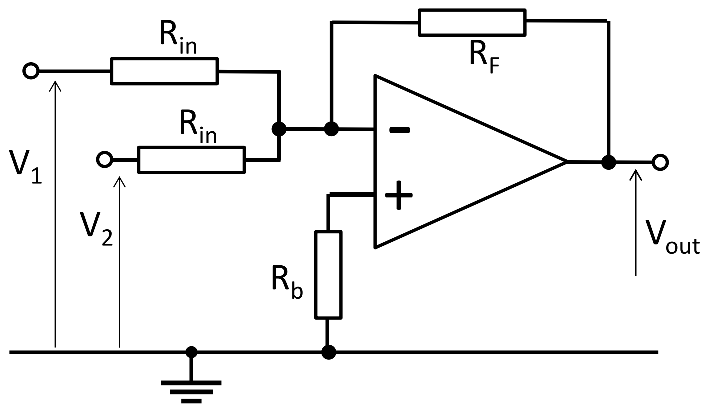


Figure 14.6 Summing amplifier with a resistor between ground and the non-inverting input

Notice the difference? There's now a resistor between the non-inverting input and ground. Why? After all, an ideal op-amp takes no current into its non-inverting input, so there will be no voltage dropped across this resistor; in other words it doesn't matter what value this resistor has, the non-inverting input will always be at ground. So why not use a zero ohm resistor (in other words no resistor at all) and save some money?

There are a couple of reasons for this, but the most interesting one[3] is that real op-amps are not ideal, and do take some current into their inputs: a very small current known as the input bias current. So a resistor between ground and the non-inverting input will have the effect of making the voltage at the non-inverting input slightly lower (or higher) than zero.

Why would you want to do that? Because there is also an input bias current flowing into the inverting input. To a first order these input bias currents can be assumed to be equal, and having a resistor between the non-inverting input and ground can eliminate the error in the output caused by these small currents. There are more details about this, and a discussion of the optimum values of this resistor, in the chapter on non-ideal operational amplifiers.

14.2.5 The non-inverting summing amplifier

Not all summing amplifiers are inverting. It is possible to make a circuit which adds together two voltages without inverting the output, although this is slightly less common in practice. The circuit looks like this:

[3] Another reason is that when testing circuits after construction, op-amps are often tested by forcing voltages onto their inputs from voltage sources, and looking at what happens to the outputs. With a non-inverting input tied directly to ground you can't force the input to anything else when testing the circuit, which means you can't test the component as fully.

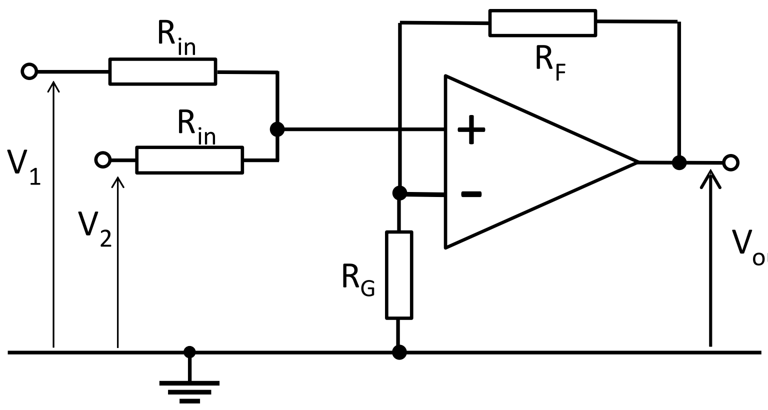


Figure 14.7 The non-inverting summing amplifier

Again, it can be analysed using superposition. With just the V1 input active and V2 connected to ground, the circuit behaves as a non-inverting amplifier with a gain of (1 + RF / RG) preceded by a potential divider made up of the two input resistors:

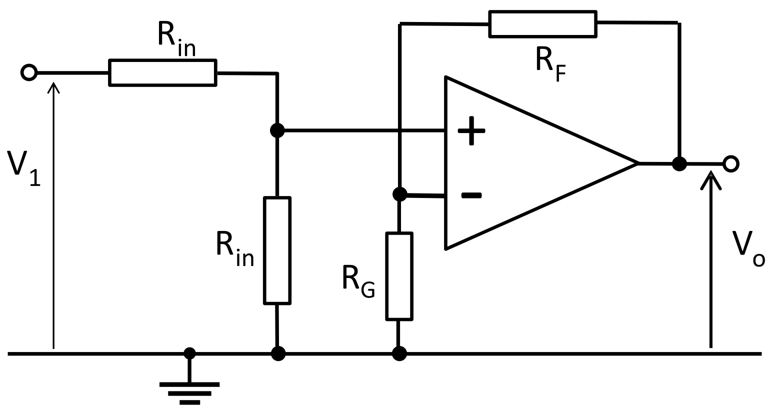


Figure 14.8 The non-inverting summing amplifier with one input grounded

This gives the output due to the first input as:

$$V_{out\_1} = \frac{V_1}{2}\left(1 + \frac{R_F}{R_B}\right) \tag{14.14}$$

and similarly for the second input. Combining these outputs using the principle of superposition gives the total output of this circuit:

$$V_{out} = \frac{(V_1 + V_2)}{2}\left(1 + \frac{R_F}{R_B}\right) \tag{14.15}$$

so that if the two input resistors are equal and $R_F = R_B$, the result is just $V_{out} = V_1 + V_2$.

With different values of input resistors on the two inputs, the non-inverting summing amplifier can output different proportions of the inputs, for example if the input resistor on V_1 is twice the inputs resistor on V_2, the result becomes:

$$V_{out} = \frac{(V_1 + 2V_2)}{3}\left(1 + \frac{R_F}{R_B}\right) \tag{14.16}$$

and with careful choice of resistors, any combination of the two inputs is possible.

There's an interesting extension of this idea: instead of two discrete resistors being used as the input resistors, a potentiometer can be used instead. (A potentiometer has a constant resistance between its ends, but a third terminal which connects to a variable place along the resistance. This provides two resistors with a common junction, and a constant total resistance, but each of the resistors can vary.) This results in the circuit below:

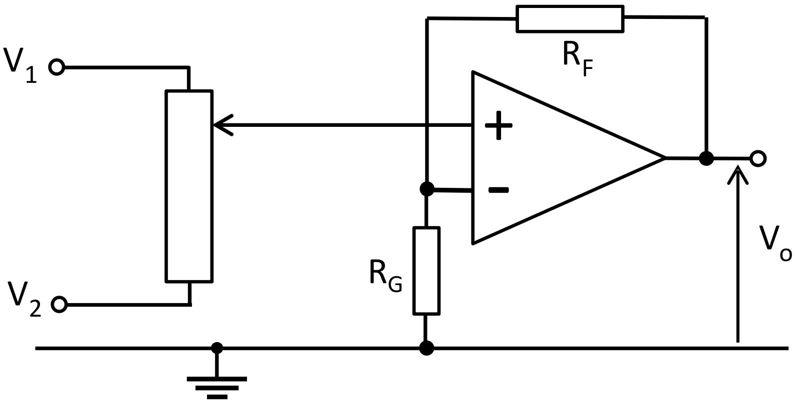


Figure 14.9 A variable mixer based on a non-inverting summing amplifier

The result is a mixer circuit, which depending on the setting of the potentiometer, can output any mixture of the inputs on V_1 and V_2.

14.3 The differential amplifier

The job of a differential amplifier is to amplify the difference between two signals. This is a little more difficult than it might sound, but it's not too hard to do. Consider the circuit shown below:

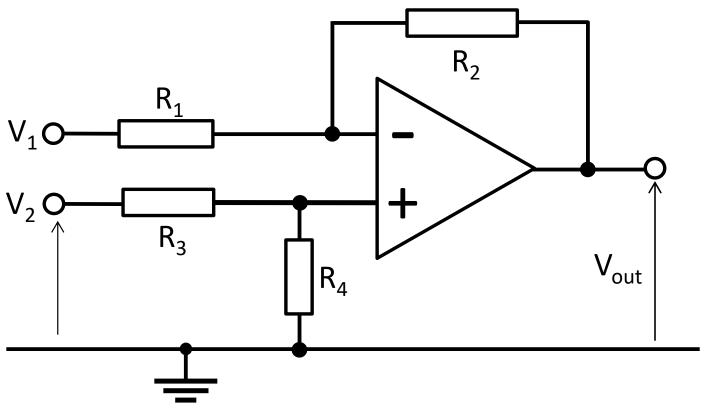


Figure 14.10 Differential amplifier configuration

Once again this is a linear circuit, so we can apply the principle of superposition, and consider each input in turn:

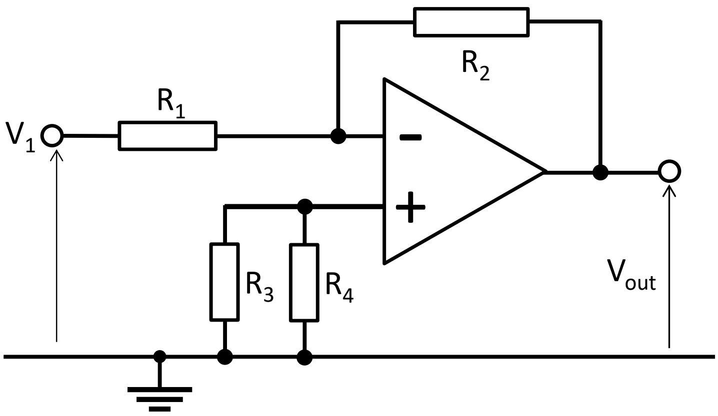


Figure 14.11 Differential amplifier with non-inverting input grounded

With just the first input (V_1) connected to an input (and the second input (V_2) connected to ground), the circuit is an inverting amplifier with a gain of $-R_2 / R_1$, so the output would be:

$$V_{out1} = -\frac{R_2}{R_1} V_1 \tag{14.17}$$

With just input V_2 connected (and V_1 connected to ground), the circuit is a potential divider, followed by a non-inverting amplifier with a gain of $\left(1 + R_2 / R_1\right)$.

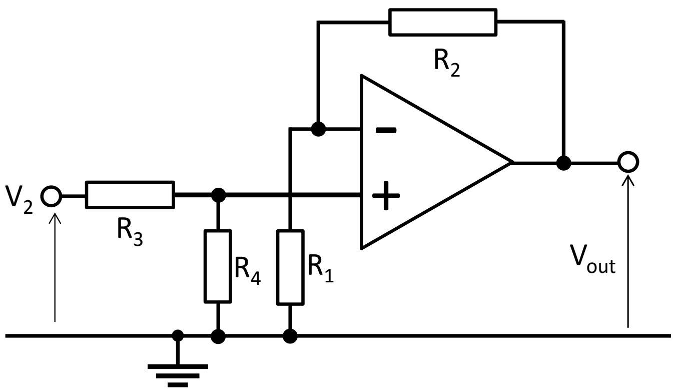


Figure 14.12 Differential amplifier with inverting input grounded

Here we have to first consider the potential divider, and note that from the standard potential divider equation, the voltage on the non-inverting input to the op-amp would be:

$$V_+ = V_2 \frac{R_4}{R_3 + R_4} \tag{14.18}$$

This is then followed by a non-inverting amplifier with a gain of $\left(1 + R_2 / R_1\right)$, so that the output of the op-amp with just V_2 being driven would be:

$$V_{out2} = \left(1 + \frac{R_2}{R_1}\right) V_+ = V_2 \left(1 + \frac{R_2}{R_1}\right) \frac{R_4}{R_3 + R_4} \tag{14.19}$$

Adding up the outputs for both inputs then produces the final output of the differential circuit, which is:

$$V_{out} = V_{out1} + V_{out2} = -V_1 \frac{R_2}{R_1} + V_2 \left(1 + \frac{R_2}{R_1}\right) \frac{R_4}{R_3 + R_4} \tag{14.20}$$

This might look rather complex, but if you ensure[4] that $R_2 / R_1 = R_4 / R_3$, then it simplifies to:

$$\begin{aligned} V_{out} &= -V_1 \frac{R_2}{R_1} + V_2 \left(1 + \frac{R_2}{R_1}\right) \frac{R_4 / R_3}{1 + R_4 / R_3} \\ &= -V_1 \frac{R_2}{R_1} + V_2 \left(1 + \frac{R_2}{R_1}\right) \frac{R_2 / R_1}{\left(1 + R_2 / R_1\right)} \\ &= -V_1 \frac{R_2}{R_1} + V_2 \frac{R_2}{R_1} \\ &= \frac{R_2}{R_1} \left(V_2 - V_1\right) \end{aligned} \tag{14.21}$$

We've designed a circuit that gives an output which is an amplified version of the difference between the two inputs. Just note that for accurate performance you do have to ensure that the ratios of the resistors are closely matched.

This circuit is extremely useful in many situations, including receiving balanced signals. So useful in fact, that you can buy more advanced versions of this circuit which have all the required resistors on the chip already, and additional input buffers on the two inputs so that negligible current is taken from the two inputs: these are known as *instrumentation amplifiers*.

14.4 Summary: the most important things to know

- Summing amplifiers output an amplified version of the sum of two inputs
  - The sum can be weighted, so that they can output, for example, $2 * V_1 + V_2$
- Differential amplifiers output an amplified version of the difference between two inputs
  - It is important for differential amplifiers that the ratio of the resistors is correct
- Both types of circuit are linear, and can be readily analysed using superposition.

[4] Not always easy to do, since it requires very accurately matched resistor values. This is one reason why designers often prefer to use specialist differential amplifier chips (rather than build their own circuit out of op-amps) when very high accuracy is required.