13 A Short Introduction to Operational Amplifiers

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Prerequisite knowledge required: Resistor Networks, Circuit Loading, Ohm and Kirchhoff's Laws

13.1 Introduction

One of the most common requirements of an electronic circuit is to make signals bigger; this is called *amplification*. This is such a common requirement that a lot of integrated circuits[1] have been developed with the sole purpose of providing amplification. The most common type of integrated circuit amplifier is the operational amplifier, of which there are literally hundreds of varieties, costing from a few pence to hundreds of pounds.

In this chapter I'll introduce the basic function of the operational amplifier, and show how they can be used to provide both voltage and current gain.

13.2 The operational amplifier

In its simplest form, the operational amplifier has five connections to the outside world: the non-inverting input, the inverting input, the output, a positive power supply and a negative power supply[2]. The circuit schematic element (along with a sketch of a typical op-amp package and the standard pin-out for an 8-pin package is shown below):

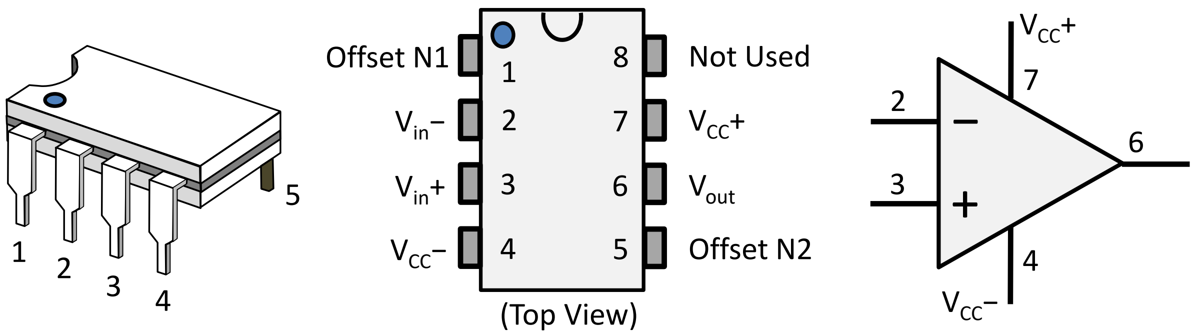

Figure 13.1 A single op-amp: sketch, top-view and schematic symbol

At low frequencies (we'll discuss what happens at high frequencies later), the op-amp shown in the figure above takes the voltage at pin 3 (the non-inverting input, V_{in+}) subtracts the voltage at pin 2 (the inverting input, V_{in-}), multiplies this difference by a very large number *A* known as the *open-loop gain* (typically 100,000 or so), and presents the output on pin 6 (the output, V_{out}).

$$V_{out} = A\left(V_{in+} - V_{in-}\right) \tag{13.1}$$

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[1] Circuits built on one piece of (usually) silicon, containing a lot of diodes, transistors, resistors, capacitors and other circuit elements, and sold mounted in a convenient (usually) plastic shell with (often) legs to allow connections to be made to the internal circuitry. See Figure 13.1 (left-hand drawing).

[2] There is no separate ground connection, although in some cases, the negative power supply pin can be connected to ground, so that the op-amp is powered from a single positive voltage supply and ground. This is known as "single-supply" or "single-rail" operation.

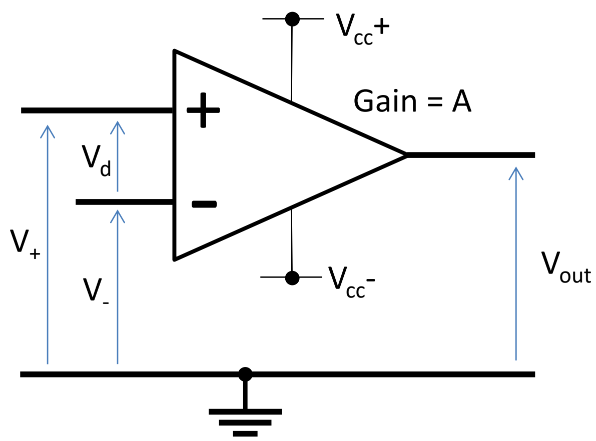

Figure 13.2 Basic op-amp operation: $V_{out} = A^* V_d$

Or at least that's what an ideal op-amp would try to do.

It might not succeed because it is impossible for an op-amp to output a voltage greater than its positive power supply or lower than its negative power supply, and most op-amps can't even do that: the output is restricted to the range between a few hundred millivolts above the negative power supply (V_{CC}-) to a few hundred millivolts below the positive power supply (V_{CC}+)[3].

Another case in which it might not succeed is when the inputs are outside the acceptable range. Only a very few op-amps can accept input signals below the negative power supply or above the positive power supply; and just like with outputs, most of them only work as long as their inputs are within a certain range between the negative and positive supplies[4].

For example: consider a typical op-amp such as a TL071 powered from +15 V and -15 V. The TL071 has a typical open-loop voltage gain at low-frequencies[5] of 200,000. Connect the inverting input (pin 2) to ground, and the non-inverting input (pin 3) to minus one volt. 200,000 times the difference would be:

$$\begin{aligned} 200{,}000\left(V_{in+} - V_{in-}\right) &= 200{,}000\left(0 - (-1)\right) \\ &= 200{,}000\left(0 + 1\right) \\ &= 200 \text{ kV} \end{aligned} \tag{13.2}$$

[3] Op-amps which can output across the whole range from the negative power supply to the positive power supply are said to have *rail-to-rail outputs*. This can be very useful when working with small supply voltages in battery-powered equipment.

[4] Op-amps which can accept inputs across the whole range from the negative power supply to the positive power supply are said to have *rail-to-rail inputs*. Having an input range which goes down to the negative supply can be particularly useful when working with single-rail circuits in which the op-amps use ground as their negative power supply.

[5] See the datasheet for the TL071 available on-line, or in the lab scripts.

but a typical TL071 cannot output a voltage any greater[6] than +13.5 V when powered from +15V and -15V, so in this case the output of the op-amp would be around 13.5 V. The op-amp is said to be in *saturation*.

There are only two other things you need to know about op-amps before you can start designing a very wide range of circuits using them. These things are:

1. They have a very large input impedance
2. They have a very small output impedance

(If you're unsure what these terms mean, refer to the chapter on Circuit Loading.)

13.2.1 Ideal op-amps

(This section is a mostly a summary of the points made in the previous section.)

When starting a circuit design, it's often useful to be able to define an "ideal" op-amp: one with a set of properties that make designing circuits with op-amps very easy. An ideal op-amp can be defined as an op-amp with the following properties:

1. The input impedance is infinite: no current ever flows into the inverting or non-inverting inputs.
2. The output impedance is zero: it doesn't matter what load is connected to the output of the op-amp, the voltage on the output will remain the same.
3. The open-loop gain is effectively infinite: this implies that in any circuit in which the op-amp is not in saturation, the inverting input and non-inverting input must be at effectively the same voltage[7].
4. Perfect differential gain[8].

For the sake of completeness, there are a few other properties of an ideal op-amp which we'll come back to later, but which aren't as important as these four for this introduction. These include:

5. Noiseless: they do not add any noise to the signals passing through them.
6. Infinite power supply rejection ratio: any variations in the voltage on the power supply pins do not cause any changes at the output.
7. Infinite bandwidth: the open-loop gain is not a function of frequency.
8. Rail-to-rail operation: the inputs and outputs can be anywhere from the negative power supply to the positive power supply.

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[6] Again see the datasheet for the TL071. Different op-amps behave differently, and note this is a typical specification only; if you tried to measure this with an op-amp in the lab you might get a slightly different answer with the individual op-amp you use for the measurement.

[7] I've used the word "effectively" here, since trying to deal with an actual infinite gain causes mathematical problems, and a truly infinite gain is impossible. So just consider the gain of the op-amp to be so large that it might as well be infinite.

[8] In other words the output is given by the open-loop gain times the difference between the voltages at the non-inverting input and the inverting input only. In real op-amps this is not the case, it's not simply the difference between the inverting and non-inverting inputs which is amplified, it's this difference plus an offset known as the input offset voltage. In many cases this input offset voltage is small, and can be neglected.

13.3 The first simple circuit: a non-inverting amplifier

Now we've got the basics, we can look at a simple practical (and very common) op-amp circuit: a voltage amplifier. Consider the circuit shown in the figure below (note I haven't shown the power supply connections to the op-amp here; this is very common shorthand, but don't forget that for any op-amp circuit to work in practice the op-amp must have a power supply connected):

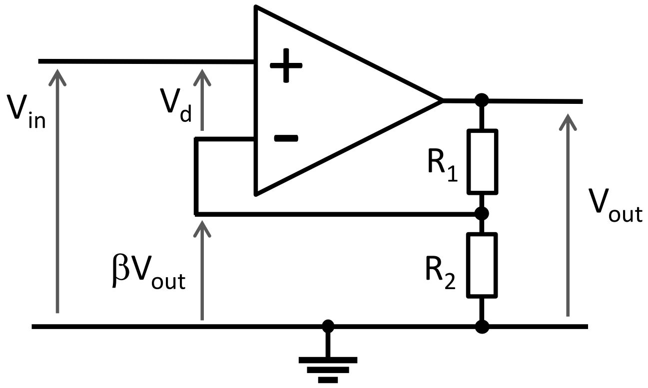

Figure 13.3 A basic non-inverting amplifier

Consider the operation of this circuit: firstly, the inverting input is connected to the output from a potential divider between the op-amp output and ground, so we could write:

$$V_{in-} = V_{out} \frac{R_2}{R_1 + R_2} \tag{13.3}$$

and the non-inverting input is connected directly to the input voltage, so:

$$V_{in+} = V_{in} \tag{13.4}$$

and substituting into equation (13.1) gives:

$$\begin{aligned} V_{out} &= A\left(V_{in+} - V_{in-}\right) = A\left(V_{in} - V_{out}\frac{R_2}{R_1 + R_2}\right) \\ V_{out} + AV_{out}\frac{R_2}{R_1 + R_2} &= AV_{in} \\ V_{out} &= \frac{AV_{in}}{\left(1 + A\frac{R_2}{R_1 + R_2}\right)} \end{aligned} \tag{13.5}$$

which suggests that the voltage gain is:

$$\frac{V_{out}}{V_{in}} = \frac{A}{\left(1 + A\frac{R_2}{R_1 + R_2}\right)} \tag{13.6}$$

Now as A is ideally infinite, $AR_2 / (R_1 + R_2)$ will be much greater than one, so we can write:

$$1 + A\frac{R_2}{R_1 + R_2} \approx A\frac{R_2}{R_1 + R_2} \qquad (13.7)$$

and therefore:

$$\frac{V_{out}}{V_{in}} = \frac{A}{\left(1 + A\frac{R_2}{R_1 + R_2}\right)} \approx \frac{A}{A\frac{R_2}{R_1 + R_2}} = \frac{R_1 + R_2}{R_2} = 1 + \frac{R_1}{R_2} \qquad (13.8)$$

So we have a circuit with a voltage gain of approximately[9] $1 + R_1 / R_2$. Since this gain is positive (in other words an increase in the input voltage produces an increase in the output voltage), this is known as a non-inverting amplifier.

While A is known as the *open-loop gain* of the op-amp, the gain of the actual amplifier built using the op-amp is known as the *closed-loop gain*. These are important terms; we'll be coming back to them later.

13.3.1 A special case: the unity-gain buffer

There's a very common special case, which is an even simpler circuit. Consider what happens if the resistor R_1 in Figure 13.3 is set to zero, and resistor R_2 is set to infinite. Effectively, we'd get this circuit:

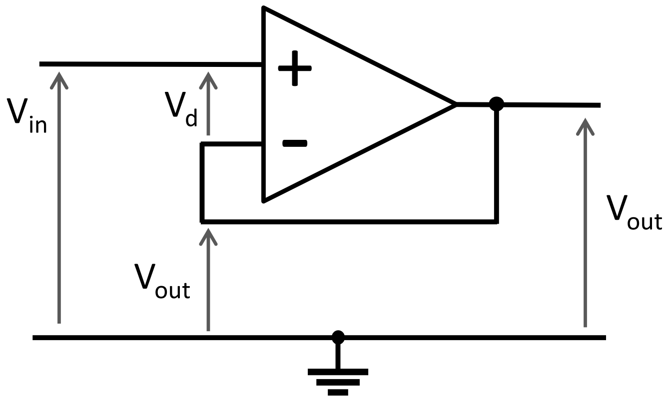

Figure 13.4 A unity-gain buffer

The equation for the gain would suggest that this amplifier has a gain of:

$$\frac{V_{out}}{V_{in}} = 1 + \frac{0}{\infty} = 1 \qquad (13.9)$$

In other words, the output is just equal to the input. (This can also be seen from looking at the circuit: for an ideal op-amp, if the output is to be finite, the two input voltages to the op-amp must be equal; and here the non-inverting input is at V_{in}, and the inverting input at V_{out}, so $V_{in} = V_{out}$.)

This is known as a unity-gain buffer. The output is equal to the input.

[9] For most op-amps (not just the ideal one), this is a very good approximation, since A really is much larger than the desired gain of the circuit (at least at low frequencies).

At this point you might be wondering what the point is of having an amplifier at all if the output is always going to be equal to the input. Consider the following circuit:

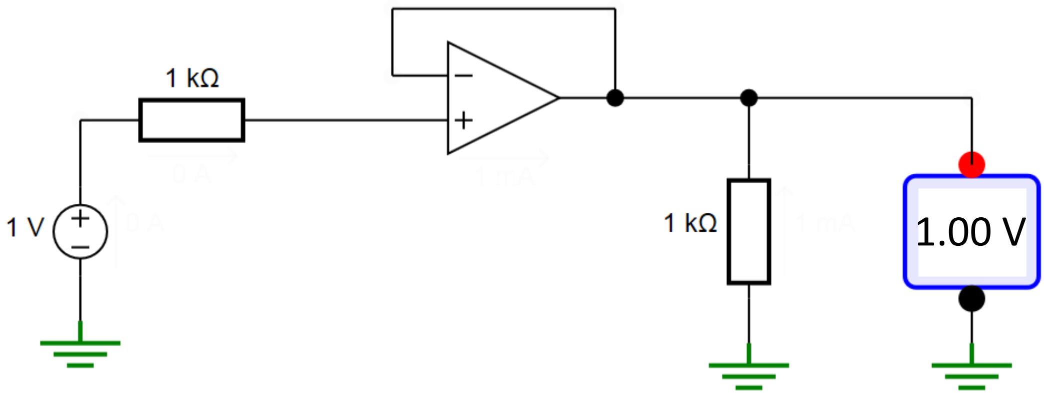

Figure 13.5 Application of a unity-gain buffer

There is no input current to the op-amp, and therefore no current through the input 1k resistor, and therefore no voltage drop across it. So the voltage at the non-inverting input to the op-amp is the same as the voltage source: one volt.

The op-amp has negligible output impedance, so the output voltage of the op-amp is not affected by the 1 mA of current flowing through the output 1k resistor, and the output voltage (as registered on the voltmeter) would be one volt.

In fact, it doesn't matter what the values of the two resistors in this circuit diagram are, the output would still be one volt. However: what if the op-amp wasn't there? The two 1k resistors were connected directly together with a wire? Now we would have a simple potential divider, and the output voltage would depend on the values of the resistors. (With the two 1k resistors shown above, the output would be 500 mV.)

Using a unity-gain buffer in this way can effectively decouple two stages of a circuit: the output impedance of the first stage (represented above by the 1k resistor at the input) and the input impedance of the second stage (represented above by the second 1k resistor at the output) no longer make any difference to the voltage received by the second stage.

13.4 A second simple circuit: an inverting amplifier

The second most useful circuit is an inverting amplifier. The basic circuit is shown in the figure below (again note I haven't shown the power supply connections to the op-amp):

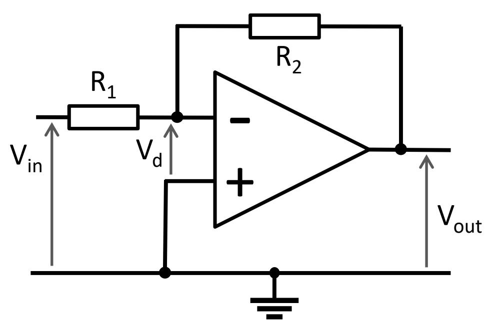

Figure 13.6 A basic inverting amplifier

The operation of this circuit is easiest to understand by thinking about the currents flowing in the circuit. The argument goes as follows:

- Assuming that the output of the circuit is within the range of the power supply (in other words the op-amp is not in saturation), the voltage difference between the two inputs (V_d) must be very small (approximately zero).
- Since the non-inverting input is tied to ground, this means that the inverting input must be very close to ground (zero volts) as well.
- Therefore the current flowing through R_1 must be $(V_{in} – 0) / R_1 = V_{in} / R_1$ by Ohm's law.
- Since none of this current can flow into the inverting input of an ideal op-amp, it must flow through the feedback resistor R_2.
- That means the voltage across the feedback resistor from left to right must be $(V_{in} / R_1) * R_2$ by Ohm's law.
- The left-hand side of the feedback resistor is at ground potential (zero volts), so the output of the op-amp must be $(V_{in} / R_1) * R_2$ below ground, in other words at $– (V_{in} / R_1) * R_2$ volts.
- Therefore the gain of the amplifier is:

$$\text{Voltage Gain} = \frac{V_{out}}{V_{in}} = \frac{-(V_{in}/R_1)\times R_2}{V_{in}} = -\frac{R_2}{R_1} \tag{13.10}$$

Note that this is negative, which implies that the amplifier inverts the incoming signal: a small increase in the input voltage would result in a decrease in the output voltage. Also note that unlike the non-inverting amplifier, the inverting amplifier circuit takes a significant current from the input V_{in}.

(You can derive the same answer by considering that the current through the two resistors must be the same, and therefore that:

$$\frac{V_{in} - V_-}{R_1} = \frac{V_- - V_{out}}{R_2} \tag{13.11}$$

and that from equation (13.1):

$$V_{out} = A(0 - V_-) \tag{13.12}$$

and then go through the maths, letting A tend to infinity, but the algebra gets a bit more tedious.)

13.5 Voltage gain and current gain

Op-amps can be used in circuits that provide both voltage and current gain (and often both).

Voltage gain is simple to define and imagine. A circuit has voltage gain if the voltage at the output is a magnified version of the voltage at the input. Since op-amps provide a voltage source at their outputs and take voltages as their inputs, this is almost always the most important thing to know.

In some circumstances, a current gain can also be defined for op-amp circuits, although it's debatable how useful this is. The current gain is the output current divided by the input current, and in many cases it's enormous. For example: consider the following circuit, where a voltage source with an internal resistance drives a resistor load through a unity-gain buffer:

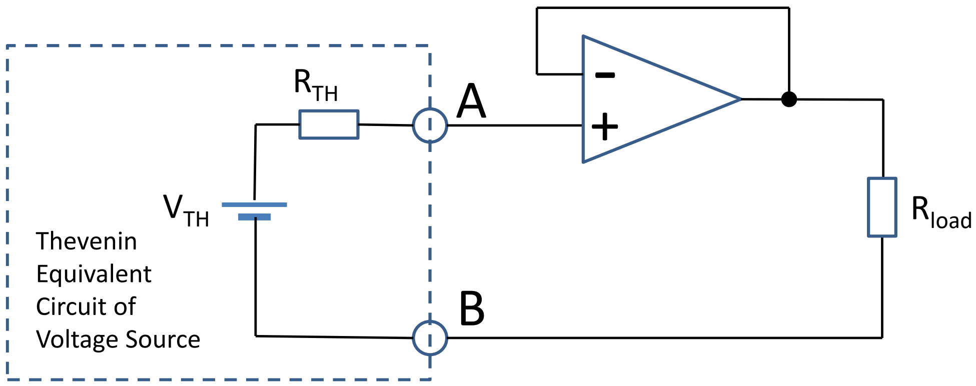

Figure 13.7 Application of a unity-gain buffer

Now in this case the voltage present at the input to the unity gain amplifier is V_{TH}, and assuming an ideal op-amp with infinite input impedance, and there is no input current to the circuit at all. The output current, however, is the current flowing into the load, which is

$$\text{Current} = \frac{\text{Voltage}}{\text{Resistance}} = \frac{V_{TH}}{R_{load}} \tag{13.13}$$

so the current gain here is infinite:

$$\text{Current Gain} = \frac{\text{Current Out}}{\text{Current In}} = \frac{V_{TH}/R_{load}}{\approx 0} \approx \infty \tag{13.14}$$

which clearly isn't very useful as a result.

13.5.1 More sensible current gains: the inverting amplifier

It's more sensible to determine the current gain when an inverting amplifier is being used. For example, consider the following circuit:

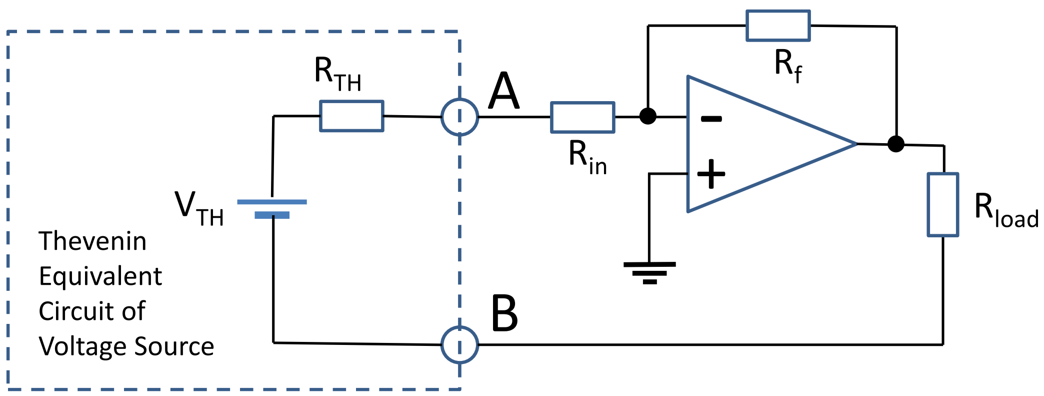

Figure 4 Application of an inverting buffer

The input current to this amplifier is the current through R_{in} down to ground[10], and the output current is the current through the load R_{load}. Both are finite and have sensible values, so here we can define a current gain:

$$\text{Current Gain} = \frac{\text{Current Out}}{\text{Current In}} = \frac{V_{TH}\left(\frac{-R_f}{R_{TH}+R_{in}}\right)/R_{load}}{V_{TH}/\left(R_{TH}+R_{in}\right)} = \frac{-R_f}{R_{load}} \qquad (13.15)$$

However it's worth repeating that determining a value for the current gain of op-amp circuits isn't very useful in most practical cases. It's just important to know that it happens.

13.6 Summary: the most important things to know

- Op-amp are incredibly useful circuit elements, they output an amplified version of the difference between the voltage at their non-inverting input and the voltage at their inverting input.
- Op-amp require a power supply to work.
- Ideal op-amps have:
    - Infinite input resistance (no current flows into their inputs)
    - Zero output resistance (the output looks like a voltage source)
    - Infinite gain at all frequencies
    - Can accept inputs anywhere between the positive and negative power supply voltages ("rail-to-rail inputs")
    - Can produce outputs anywhere between the positive and negative power supply voltages ("rail-to-rail outputs")
    - Have perfect differential gain (the output voltage is = $A(V_+ - V_-)$)
- A non-inverting amplifier (see Figure 13.3) has a voltage gain of $1 + R_f / R_b$
    - where R_f is the feedback resistor between the output and inverting input, and
    - R_b is the resistor between the inverting input and ground.
- An inverting amplifier (see Figure 13.6) has a voltage gain of $-R_f / R_{in}$
    - where R_f is the feedback resistor between the output and inverting input, and
    - R_{in} is the resistor between the input and the inverting input.

[10] Remember that if the op-amp is not saturated, the voltage on the inverting input must be approximately the same as the voltage on the non-inverting input, which is tied to ground here.