9 A Short Introduction to Electric and Magnetic Fields

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Prerequisite knowledge required: Newtonian Physics

9.1 Introduction

In this chapter we take a step back from thinking about voltage in terms of a sort of pressure on charge and start to ask more fundamental questions like "how does a charged particle know which way to go?" and "why does Kirchhoff's voltage law work?". To answer these questions we need to go back to the earliest experiments into electric phenomena, and consider a theory that can explain the observed force between two charged particles and how this force actually gets through space.

Which if you think about it, isn't obvious. How does a charged particle know that there is another charged particle somewhere close to it?

Once developed and understood however, this theory has some other very interesting consequences, including allowing us to predict the capacitance of a capacitor based on its physical dimensions, and extensions to the theory predict radio waves and light.

First a bit of terminology: a "field" in the sense I'm using it here is a parameter which has a particular value at every point in space. You could, perhaps, think of the temperature in a room as a field since a value of temperature could be measured everywhere[1]. However, a closer analogy to what I want to talk about here would be gravity: the force of attraction between every two objects which have mass. Anywhere around any given mass, a small unit mass would feel a force of attraction, and that force has a different direction and magnitude depending on where the small mass is. That makes the gravitational force of attraction a field as well[2].

Electric and magnetic forces have a lot in common with gravity, so this is probably a good place to start.

9.2 The gravity analogy

Any object with any mass experiences a gravitational force pulling it in a certain direction. This force is the sum of all of the forces resulting from all the other objects in the Universe. (It might not appear that way in everyday life, but that's because one object (the Earth itself) is so huge and close that its gravitational influence swamps everything else.)

In the case of gravity, there is a simple equation, experimentally derived, which determines the size of this force:

[1] It's more likely to have higher values towards the ceiling since hot air rises, but it does have a value at every point.

[2] You might notice a subtle but important distinction between these two fields. Temperature is just a value, it doesn't have a direction associated with it. That makes temperature a *scalar field*. The gravitational force does have a direction, so that makes gravitational force a *vector field*. We'll come back to these terms later.

$$F = G\frac{m_1 m_2}{r^2} \tag{9.1}$$

Here F is the gravitational force (measured in Newtons), m_1 and m_2 are the masses of the two bodies (in kilograms), r is the distance between them (in metres), and G is a constant. Note that this is an "inverse-square" law: the force varies with the inverse of the square of the distance between the objects; for example double the distance and the force gets smaller by a factor of four.

9.2.1 Energy in the gravity analogy

It might not be immediately obvious, but there is energy stored in the gravitational field.

Assume you have two objects of masses m_1 and m_2 a certain distance r apart, and you move them a little bit further apart, perhaps by a distance dr. Since there is a force of attraction between them, you will have to do work (input some energy into the system) to move them further apart (just like picking something up off the floor, when you have to do work to increase the distance between the thing and the Earth).

What happens to that energy? Energy is conserved, so it must go somewhere. The answer is that it is stored in the gravitational field. It can be released, for example, by letting go of one of the objects so that it's free to accelerate towards the other one: in this case the energy comes out of the gravitational field and turns into kinetic energy in the mass.

How much work is required to move the masses apart? Work is just force times the distance, so the small amount of work δW required to move the two objects a small distance δr further apart is:

$$\delta W = F \times \delta r = G\frac{m_1 m_2}{r^2}\delta r \tag{9.2}$$

How about the work required to move them an infinite distance apart? We can work that out by combining all the small amounts of work required to move one of them all the small distances dr all the way from r to infinity, and that can be done by integrating the above equation to get:

$$\text{Total Work} = \int_0^W dW = \int_r^\infty G\frac{m_1 m_2}{r^2}dr = Gm_1 m_2\left[\frac{-1}{r}\right]_r^\infty = G\frac{m_1 m_2}{r} \tag{9.3}$$

Now think about things the other way: suppose you started at infinity, and allowed the objects to move closer together until they were a distance r apart. If it takes Gm_1m_2/r joules of work to get them an infinite distance apart, then the field must release the same amount of energy when they are brought back to a distance r apart again.

Where does that energy go? Well, if you can imagine an infinite universe completely empty except for these two objects, and you let them go, they would start to move closer together by themselves, and accelerate as they got closer together and the force between them got stronger. The energy released from the gravitational field would become kinetic energy.

9.2.2 The concept of potential

We can make the calculations of the energy gained and lost by masses moving in a gravitational field at lot easier by developing the idea of a *potential*. The gravitational potential of a location in space is the work that has to be done to get a unit mass from a chosen reference position (in this case an infinite distance away) to that point in space[3]. From the above equation, for the case of this unique universe in which there are only two objects, this can be readily seen to be:

$$\text{Potential} = -G\frac{m_1}{r} \tag{9.4}$$

The negative sign is here because the potential is the amount of work that needs to be done to get the mass m_2 (now set to be the unit mass, so now $m_2 = 1$) from infinity to a distance r away from the mass m_1, and that's a negative energy: energy is released in this process, you don't have to put in any work to achieve this, any more than you have to do any work to make something fall down onto the floor. Energy is released from the mass in this process: it is moving in the direction of the force.

If we wanted to know how much work was required (or energy released) to get a non-unit mass m_2 from one place to another, all we have to do is find out the difference in potentials between these two places, then multiply this by the mass m_2.

For example: in this strange "only two masses exist anywhere" universe, how much work is required to move a mass of m_2 from a point a distance R_1 from mass m_1 to a point a distance R_2 from the mass m_1?

As before we could do this by integration:

$$\text{Energy Required} = \int_{R_1}^{R_2} G\frac{m_1 m_2}{r^2}dr = Gm_1 m_2\left[\frac{-1}{r}\right]_{R_1}^{R_2} = Gm_1 m_2\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \tag{9.5}$$

but it's much easier just to use the concept of potential, and note that the difference in potential between these two points is[4]:

$$\text{Potential Difference} = \frac{-Gm_1}{R_2} - \frac{-Gm_1}{R_1} = Gm_1\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \tag{9.6}$$

so the difference in energy between an object of mass m_2 at the two points (and which must therefore be equal to the work required to move the object between those points) is:

[3] This is the same use of the word "potential" as in "potential energy". The "potential energy" of a mass is the energy it has as a result of just being at certain point in space.

[4] Note that this implies that it takes no energy at all to move the objects around each other, provided the distance between them is kept constant. This is indeed true. It would take some energy to get one of the objects to start moving (it will need some kinetic energy if it's going to move), but all of this energy could, in principle, be reclaimed by removing its kinetic energy once it gets to its new location.

Consider a planet in a circular orbit around its sun: the distance isn't changing so the gravitational potential isn't changing either. The planet requires no external energy to continue moving around its sun.

$$\text{Energy Required} = Gm_1\left(\frac{1}{R_1} - \frac{1}{R_2}\right)m_2 \quad (9.7)$$

No integration required. This is one reason why the concept of a potential is very popular. However the idea is even more useful than this: consider a universe with more than two objects in it…

9.2.3 What about a universe with more than two objects in it?

I'm glad you asked. If there are a large number of objects with mass in the Universe, then the total gravitational force an object feels is the sum of all of the individual gravitational attractions to every other mass[5]. That's quite straightforward in principle, but the result can be difficult to calculate since all the forces are pulling in different directions.

What is not quite so obvious, but is much more useful, is the fact that the gravitation potential at a point in space is the sum of the gravitational potentials due to all of the objects in the universe. This is an example of the *principle of superposition*, and it follows from the fact that the force between two objects is a linear function of the mass of the objects: double the mass of an object, and you double the potential due to that mass.

Once we know the gravitational potential at any two places, it's easy to calculate the work required (or energy released) when a mass is moved from one place to another: just multiply the mass by the difference in the potentials at the two places. There's no need to combine all the forces pulling in different directions.

9.2.4 The concept of flux

There are actually two different fields associated with gravity, and it's important to understand the distinction between them.

One of them is associated with the force: it's the force that a unit mass would experience at any point in space. This has both a magnitude and a direction[6] (the direction in which the unit mass wants to move). This is what we normally refer to as the gravitational field.

The other is associated with potential, and it's the energy that would be required to bring a unit mass from an infinite distance away to each point in space. This has only a value; there is no direction associated with a potential[7].

There is a very useful related quantity known as *flux*. Flux is a theoretical concept that can be pictured as flowing through space and provides a measure of how strong the first type of field is at any point: the more flux, the stronger the field (so the greater the gravitational force on a unit mass). Quantitatively, the flux flowing through any small area of space perpendicular to the

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[5] Do note that you have to take the direction of the forces into account when combining them: for example an object mid-way between two objects of equal mass feels no net gravitational force at all, because the forces pulling it in each direction cancel out. For those who have come across the concepts of vectors, you can treat the forces as vectors, and do vector addition to work out the resultant force.

[6] For those who have come across vectors: this makes it into a *vector field*, since there is a vector associated with every point in space: the length of the vector is the magnitude of the force, and the direction of the vector is the direction of the force on the unit mass.

[7] For those who have come across vectors and scalars, this makes it a *scalar field*.

direction of the field lines is the product of the small area and the force on a unit mass somewhere in that area.

That might be a little difficult to imagine. Consider Figure 9.1 below. The flux can be pictured as represented by *flux lines*, winding through three-dimensional space. Some of them will pass through a small area δA centred at point P, and if this area is small enough, then all of the flux lines passing through it will be parallel.

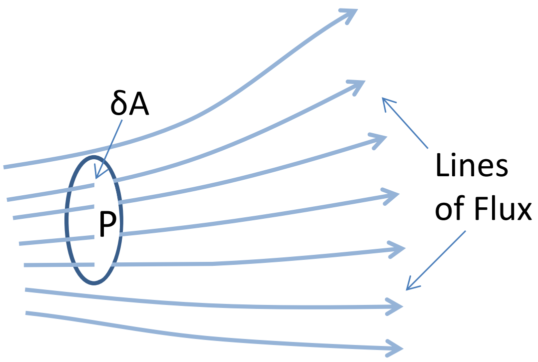

Figure 9.1 Lines of flux around a small area

The theory says that the force on a unit mass at point P will be equal to the flux density at point P (in other words, the number of flux lines passing through the small area δA divided by the area δA). You could also think of this as the distance between the lines of flux: the closer they are bunched together, the greater the force.

One reason flux is such a useful concept is that it allows us to draw diagrams showing how the force varies at each point in space. This is done using these *flux lines*, where the closer the lines of flux are bunched together, the stronger the force is at that point, and the direction of the lines of flux gives the direction of the force.

For example, the lines of flux around a single large mass would look like this[8]:

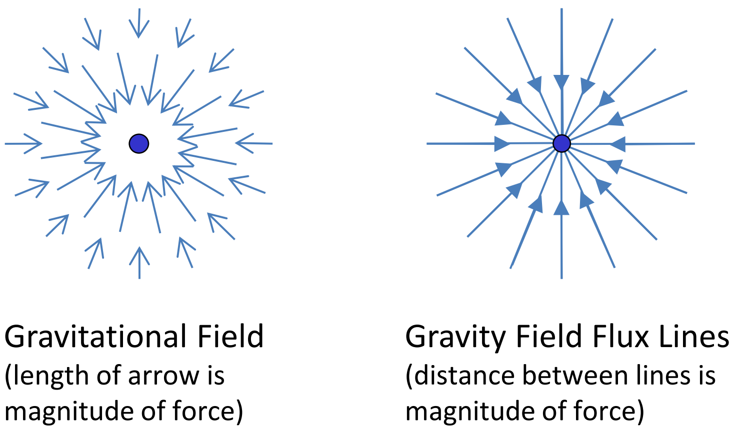

Figure 9.2 Gravitational flux lines and force lines

[8] Due to limitations of this medium (and my lack of ability as an artist), this is a two-dimensional representation only: in practice the lines of flux would emerge in all directions in three-dimensional space.

There's an important result that can be introduced at this point. Consider the question: if you add up the flux passing though the surface of a sphere centred on a point mass, what do you get?

Since the sphere is centred on the mass, all the lines of flux will be emerging perpendicular to the surface, so we can use the definition above for any small element of the area of the sphere: the flux through the small area is equal to the force on a unit mass times the small area δA:

$$\text{Flux through } \delta A = G\frac{m_1}{r^2}\times\delta A \tag{9.8}$$

and since the total surface area of a sphere is $4\pi r^2$, the total flux through the sphere must be:

$$\text{Total Flux} = G\frac{m_1}{r^2}dA\times\frac{4\pi r^2}{dA} = 4\pi G m_1 \tag{9.9}$$

which you'll note is a constant: it's not dependent on the radius of the sphere. This result holds true for any force which decays proportional to the square of the distance (so called *inverse-square-law forces*): the total amount of flux is not a function of how far you are from the mass. This is why lines of flux for these fields are continuous, and can be drawn moving ever outwards into space, with no new lines of flux starting or stopping anywhere (except at the mass m_1).

9.3 Back to electronics

Just like objects with mass and gravity, any two charged objects experience a force between them as well. This force has also been measured experimentally[9], and found to be given by:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} \tag{9.10}$$

where q_1 and q_2 are the charges, *r* is again the distance between the charges (this is another "inverse-square" law) and ε_0 is a constant, known as the permittivity of free space[10]. However it's important to note that unlike gravity, where two positive masses attract each other, two positive electric charges will repel each other.

Just like with gravity, a potential can be defined as the amount of energy required to move a unit charge from infinity to a distance *r* away from a charge q_1, this time giving:

$$\text{Potential} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r} \tag{9.11}$$

and just like in the case of gravity, this method of calculating the energy released and work required to move charges around electronic circuits is usually easier to work with.

[9] The relationship was first established by Charles-Augustin de Coulomb, and has become known as "Coulomb's Law". To recognise his achievements, the SI unit of charge is named a Coulomb.

[10] This equation holds true in free space only. If there is any material between the charges, the force can change. There's more about this effect later in the note about electromagnetism.

Note that the potential is positive in this case, because two positive electrical charges repel each other, so you have to do work to get this charge from infinity to a point closer to the other charge, and therefore the lines of flux go the other way, streaming out from a positive charge.

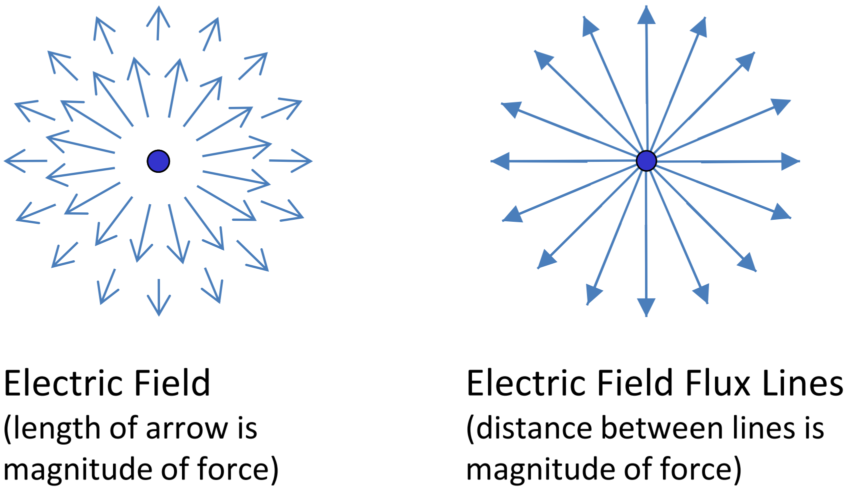

Figure 9.3 The electric field and electric flux lines from an isolated positive charge

9.3.1 Gauss's law

You might be wondering why there's a factor of 4π in the formula for the force due to an electric field, but not in the formula for the force due to gravity. (In electrostatics the constant is $1/4\pi\varepsilon_0$, whereas in gravity it was just G.)

It's because by the time they had discovered electric charge, scientists had realised that putting that factor of 4π in the constant makes the formula for the total flux emerging from the surface of a unit sphere centred on a charge q:

$$\text{Total Flux} = \frac{q}{\varepsilon_0} \qquad (9.12)$$

which is simpler, and further, that there was a way to extend this formula to arbitrary shapes, not just spheres with the charge at the centre[11]. This became known as Gauss's law: the total field strength emerging from any given volume can be simply related to the amount of charge inside the volume.

9.3.2 Dipoles

So far, I've just been writing about the field caused by one mass (or one charge). Something very important and interesting happens if two equal but opposite charges are brought close together. This is something that cannot happen with gravity (since there's no such thing as an object with negative mass) but it's easy to do with charges. The result is known as a dipole.

[11] The details of how to do this in the general case will have to wait until after you've covered vector areas and surface integrals. For now, we can derive all the results we need just from considering some special cases where the surface areas of interest are always perpendicular to the field lines.

The field due to an electric dipole is easy to calculate from superposition: just add the fields from the two charges. Similarly, you can determine the force on a small charge due to a dipole by just combining the forces due to the two individual charges that make up the dipole; however in both cases you have to be careful to ensure you take the directions of the forces into consideration when combining them.

At this point you might be asking the very reasonable question: if a dipole consists of a positive charge and a negative charge very close together, would they not just cancel each other out? Would the potential due to one not just be equal and opposite to the potential due to the other? The answer to that one is "not quite, because they are not quite at the same point in space, they are a small distance apart" (see Figure 9.4).

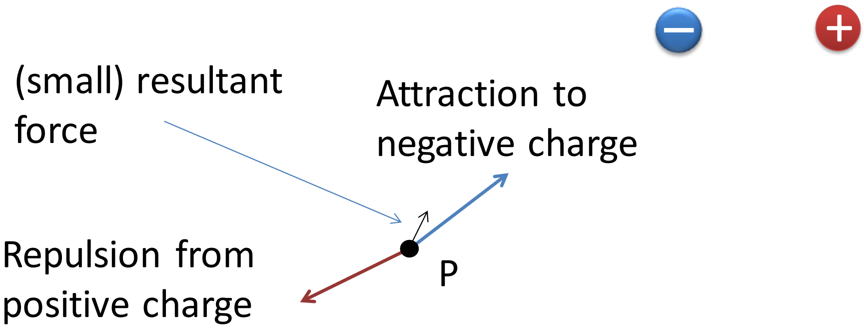

Figure 9.4 Forces on a positively charged particle P close to a dipole

Applying this technique all around the dipole results in a diagram showing the flux lines giving the direction and magnitude of the forces all around the dipole. The result is shown in Figure 9.5.

A few points to note about these diagrams:

- In the plane of all points equidistant from the two charges (shown as a dotted line in the two-dimensional drawing below), all the flux lines are perpendicular to this plane. This means that a small charge held the same distance from each charge would start to move perpendicular to this plane.
- All the lines of flux that leave the positive charge eventually end up arriving at the negative charge.
- A long distance from the dipole, the field strength is much smaller than that from a single isolated charge. This is because the further you get from the dipole, the more the two point charges appear to be in the same direction and a similar distance away, so the more exactly the two forces pull in exactly opposite directions, and hence cancel each other out.

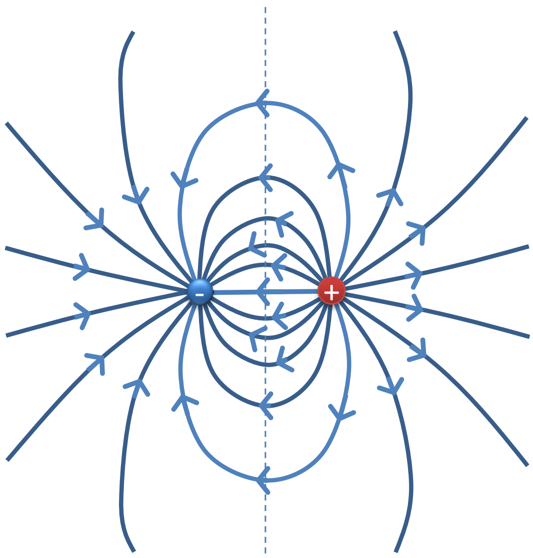

Figure 9.5 Fields around a dipole with flux lines shown

9.3.3 But electronic circuits are more than one or two isolated charges in empty space

Good point. They are vastly more complicated than this. However, they are still composed of charges moving around in space, and if you add up the potentials due to each charge, you can deduce a value of electric potential at any point in space.

By "any point in space" I'm including inside wires, batteries, resistor and capacitors as well as outside them in the surrounding space. There is an electric potential everywhere, it's just that the charges are free to move to and from some points in space (for example if there is a wire going between them), but not to and from others. It tends to be more interesting to consider the difference in electric potentials (usually shortened to "potential difference") between two points inside a circuit, as in this case the charges can move from one place to the other, so currents can start to flow and the system can actually do something useful.

9.3.4 An infinite plane of constant charge density

Imagine you have an infinite plane of constant current density, and you introduce a small charge somewhere just above the plane. What is the total force acting on the particle? This turns out to be a useful approximation to several real cases of interest.

You could work this out by integrating the forces from all the points of the infinite plane, but there's a quicker way: use Gauss' law.

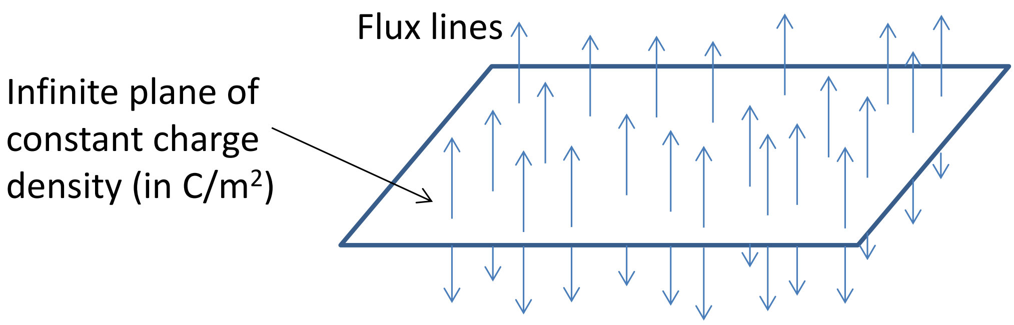

Figure 9.6 Lines of flux around an infinite plane of constant charge density

Consider the rather amateurish drawing in Figure 9.6. By symmetry, all of the field lines from an infinite plane must be streaming out perpendicular to the plane, and they must all be the same length (since on an infinite plane, every point is equivalent to every other point and every direction is equivalent to every other direction). Again by symmetry, the fields on one side must be the same as the fields on the other side of the plane.

Then, consider a small cylinder perpendicular to the plane, with a small cross-sectional area of δA as shown in figure below:

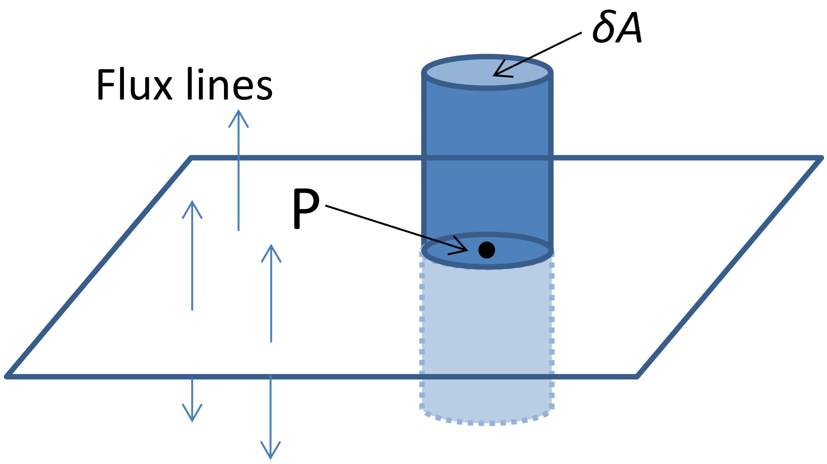

Figure 9.7 A small cylinder at a point P in an infinite plane of charge

All of the flux lines will be emerging from the ends. The total charge contained within the cylinder is $\delta A\,\rho$ where ρ is the charge per unit area on the plane and δA is the cross-sectional area of the cylinder, so the total flux emerging from the cylinder must be (from (9.12)):

$$\text{Total Flux emerging from cylinder} = \frac{\delta A\,\rho}{\varepsilon_0} \tag{9.13}$$

and since all this flux must be emerging from a total area of $2dA$ (the top and bottom ends of the cylinder), the field strength (which is equal to the flux per unit area) must be given by:

$$\text{Field strength = Flux per unit area} = \frac{\delta A\,\rho}{2\,\delta A\,\varepsilon_0} = \frac{\rho}{2\varepsilon_0} \tag{9.14}$$

Note that this is not a function of the distance away from the plane: it doesn't matter how far the infinite plane is away, the force experienced by a unit charge is always given by $\rho/2\varepsilon_0$ and acts in a direction perpendicular to the plane.

9.3.5 Two infinite planes of constant charge density

This is the last case to consider for now. Again it might seem like an impossible and artificial case, but it turns out to be a good approximation to some cases of real practical interest, particularly in the analysis of capacitors.

Here, we're assuming we have two infinite planes of charge of equal magnitude, one positive and one negative, both parallel to each other so that they never touch. Once again we can use Gauss's law to simplify the analysis, and once again we can use symmetry to quickly conclude that all the flux lines and forces must be perpendicular to the two plates, and contained with the plates.

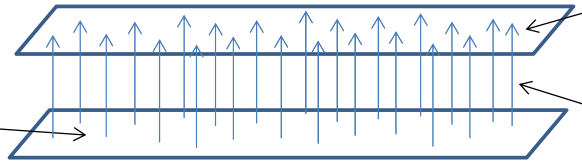

Figure 9.8 Flux lines between two infinite planes of constant and opposite charge density

First, consider a small charge anywhere outside the two planes. Since the force on a charge is not a function of the distance from an infinite plane of constant charge density, the force of attraction to the negatively charged plate must be equal and opposite to the force of repulsion from the positively charged plate. In other words, there is no force on the small charge anywhere outside the planes, which means no flux lines and zero potential. Everything of interest is happening in the region between the two planes.

Now consider a small charge anywhere between the two planes. Here the force of attraction to the negatively charged plate will be pulling in the opposite direction to the force of repulsion from the positively charged plate, so the two forces will add up. It doesn't matter where the unit charge is (the forces are not a function of distance from the planes). The result is a uniform electric field which is the same everywhere between the two planes.

The force on a unit charge here is twice the force exerted by just one of the plates, which makes it:

$$\text{Force on small charge } q \text{ between the plates} = 2 \times \frac{\rho}{2\varepsilon_0} \times q = \frac{\rho q}{\varepsilon_0} \tag{9.15}$$

and the electric field strength is the sum of the electric fields strengths from both plates (given by equation (9.14)), which gives:

$$\text{Electric field between the plates} = 2 \times \frac{\rho}{2\varepsilon_0} = \frac{\rho}{\varepsilon_0} \tag{9.16}$$

9.3.5.1 Capacitors

This, essentially, is how capacitors are made: with two parallel plates separated by a small distance. Since the electric field is given by equation (9.16), a simple formula for capacitance can be derived.

In this case the electric field between the plates is constant, so the difference in the potential (the voltage) between the two plates can be determined using:

$$\text{Voltage between the plates} = \int E\,dx = E\int dx = \frac{\rho}{\varepsilon_0}\int dx = \frac{\rho}{\varepsilon_0} d \tag{9.17}$$

where d is the distance between the two plates. If the area of the plates is much larger than the distance between them, then the field lines look almost the same as infinite plates would with the same charge density. If the area of the plates is A, then the total charge on the plates is:

$$\text{Charge on each plate} = Q = A \times \rho \tag{9.18}$$

and substituting this into equation (9.17) gives:

$$V = \frac{Q}{A\varepsilon_0}d \tag{9.19}$$

which can be re-arranged to give:

$$Q = \frac{\varepsilon_0 A}{d}V \tag{9.20}$$

which when compared to the well-known formula for capacitance:

$$Q = CV \tag{9.21}$$

implies that for a parallel-plate capacitor:

$$C = \frac{\varepsilon_0 A}{d} \tag{9.22}$$

(It's worth noting at this point that this is true only when there is nothing between the two plates. If there is something between the plates (which there almost always is in practice), the formula becomes:

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d} \tag{9.23}$$

where ε_r is the relative permittivity of the medium between the two plates. See the chapter on Electrons in Solids for more about the properties of dielectric materials and how they arise.)

9.3.6 Energy in electric fields

Just like in gravitational fields, energy can be stored in electric fields. Consider two positive charges, force them together, and then release them. They will experience a repulsive force and start to move apart. If they start to move, they must be acquiring kinetic energy. However energy is conserved… so where does that energy come from?

Again the answer is that it comes from the field itself. You can store potential energy in electric fields just like storing potential energy in springs. It's useful to quantify this: how much energy is stored in an electric field? The answer is that the energy stored per cubic metre is:

$$Energy / m^3 = \frac{\varepsilon_0 E^2}{2} \tag{9.24}$$

where E is the electric field strength. Where does this formula come from? It's probably easiest to consider the case of those two infinite parallel plates of opposite charge densities.

Consider the two plates as shown in Figure 9.8 and move them a small distance δr further apart. How much energy is required to do this? The answer is an infinite amount, because they are infinitely big plates, so we're having to move an infinite amount of charge. Not very useful. So I'll rephrase the question: how much energy is required per unit area of the plates?

Consider a unit area of the plates: this will contain a charge $+\rho$ on the positive plate, and a charge of $-\rho$ on the negative plate. Then consider moving the negative plate a small distance away from the positive plate[12]. The total force of attraction of all the charges in the unit area on the negative plate will be:

$$\text{Force per unit area} = \text{Field} \times \text{charge per unit area} = \frac{\rho}{2\varepsilon_0} \times \rho = \frac{\rho^2}{2\varepsilon_0} \tag{9.25}$$

The energy required for this unit area to move against a force is:

$$\text{Energy per unit area} = \text{Force per unit area} \times \text{distance} = \frac{\rho^2}{2\varepsilon_0} dr \tag{9.26}$$

so this must be the additional energy stored in the field per unit area of the plates. Note that again this is not a function of the distance between the plates. It doesn't matter how far the plates are apart, it takes the same amount of energy to move them a small distance dr further apart.

We can now work out the total energy stored in the field. Imagine we started with both plates together, so there is no net charge anywhere. There would be no electric field, and no energy stored. Then if it requires an energy given by (9.26) to move the each unit area of the plates a distance dr apart, and the plates end up a total distance of d apart, the total energy required, per unit area of the plates is:

$$\text{Energy in field per unit area of plates} = \int_0^d \frac{\rho^2}{2\varepsilon_0} \delta r = \frac{\rho^2}{2\varepsilon_0} d \tag{9.27}$$

This is the energy associated with just one unit area. We can go further, and note that the volume of the field (the volume with a cross-sectional area of one and a length of d) is $V = Ad$ which is just d here since the area A is a unit area, and hence:

$$\text{Energy in field per unit volume} = \frac{\frac{\rho^2}{2\varepsilon_0} d}{V} = \frac{\frac{\rho^2}{2\varepsilon_0} d}{d} = \frac{\rho^2}{2\varepsilon_0} \tag{9.28}$$

[12] This is an arbitrary choice; we could just as well have chosen to move the positive plate, the maths would all work out exactly the same.

This is known as the energy density of the field. Since we know from equation (9.16) that the field between the two plates $E = \rho / \varepsilon_0$, we can write this in the more usual form:

$$\text{Energy in electric field per unit volume} = \frac{\varepsilon_0 E^2}{2} \tag{9.29}$$

That's an equation worth remembering. It works no matter what shape the electric field is, not just between infinite parallel plates.

9.4 On to magnetism

Magnetism is slightly different, since there is no such thing as an isolated magnetic charge. They just don't exist. However, undaunted, we can still define a magnetic potential based on the energy that would be required to move an isolated magnetic charge around if we ever find one.

At this stage you might be wondering how, if magnetic charges don't exist, can you ever generate any magnetic fields? The answer is that it's only *isolated* magnetic charges that don't exist. We can use all the same theory by making the assumption that magnetic charges do exist, it's just that they always go around in pairs (one positive and one negative) in dipoles (this is known as the *Gilbert model*). To work out the magnetic potential at a point, you have to add up the potentials due to both of the magnetic charges in the dipole, and the shape of the flux lines looks just like that for the electric dipole, with force lines running from the positive to the negative poles of the magnet.

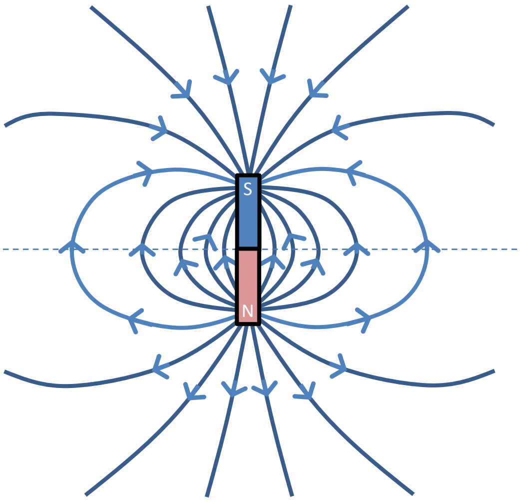

Figure 9.9 Flux lines and force lines around a dipole

I should also start using the more usual terminology at this point: instead of "positive" and "negative" magnetic charges, it's more usual to talk about "North poles" and "South poles".

Using this Gilbert model, we can define a magnetic force, where the equivalent to Coulomb's law states that:

$$F = \frac{\mu_0}{4\pi} \frac{s_1 s_2}{r^2} \tag{9.30}$$

where s_1 and s_2 are the strengths of the two magnetic poles. The constant μ_0 is called the *permeability of free space.*

At this point things might start to seem a bit more familiar: you are probably aware that if you have two magnets (with a North pole (or positive charge) at one end and a South pole (or negative charge) at the other), then like poles repel each other, but unlike poles attract.

This is exactly the same as with electric charges: like charges repel each other, unlike charges attract each other.

However since magnetic charges always go around in dipole pairs, it's perhaps not quite so obvious why magnets attract each other. Consider the two dipoles in :

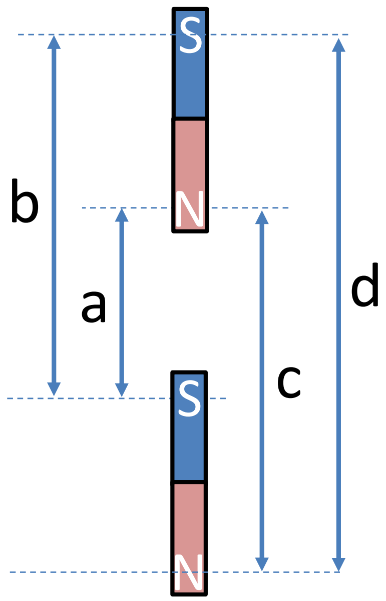

Figure 9.10 Two magnetic dipoles attracting each other

Assuming that this situation can be modelled with magnetic charges of equal magnitude s at all four ends of the two dipoles, the resultant force is given by:

$$F = \frac{\mu_0 \left| s^2 \right|}{4\pi} \left(\frac{1}{b^2} + \frac{1}{c^2} - \frac{1}{a^2} - \frac{1}{d^2} \right) \tag{9.31}$$

It's an interesting exercise (left here for the reader) to prove than in the case illustrated here (where b = c and d = a + b) this always gives a negative result, and hence a force of attraction between the two magnets.

9.4.1 Energy in magnetic fields

You can store energy in magnetic fields too. In this case, in free space the relevant formula is:

$$Energy\,/\,m^3 = \frac{B^2}{2\mu_0} \tag{9.32}$$

where μ_0 is the permeability of free space, and B is the magnetic field (measured in tesla).

9.5 Summary: the most important things to know

- A "field" is a quantity which has a different value at every point in space.
- The "potential" of a point in a space is the total energy required to move a unit charge from an infinite distance away to that point.
- If the potential of point A is higher than that of point B, then energy is released when a positive charge moves from A to B, and work is required when a negative charge moves from A to B. (The exact opposite is true when the potential at point A is lower than that at point B.) I'll summarise in a table:

Table 9-1: Moving a charge from A to B

Potential at A	Potential at B	Positive charge	Negative charge
Higher	Lower	Energy Released	Work Required
Lower	Higher	Work Required	Energy Released

- The amount of energy required (or released) is the product of the potential difference and the charge: energy = charge * voltage. (If you consider a lot of charge moving at a constant rate (i.e. a constant current) between two points a fixed potential difference apart, and therefore energy being used at a constant power rate, we get the more useful equation power = current * potential difference.)
- Therefore the "potential difference" between two points in space is the energy required to move a unit charge between those two points.
    - The "voltage" of a point is the potential difference between that point and the electric potential at a reference point defined to be at zero (known as "ground").
    - The net energy required to move a unit charge from A to B and then back again is zero, which is where Kirchhoff's voltage law comes from.
- For both electric and magnetic fields, like charges repel each other, unlike charges attract.