8 A Short Introduction to DC Circuit Analysis

v1.4 – June 2021

Prerequisite knowledge required: Ohm's Law and Kirchhoff's Laws, Resistor Networks, Linearity and Superposition

8.1 Introduction

Circuit analysis is the process of deducing the voltages at every node and the currents flowing through every component in a circuit, from knowledge of the values of all the components and the topology of the circuit. It's perhaps the most fundamental skill in analogue electronics, and vital for circuit design.

We've seen a few simple examples already. For example, the potential divider:

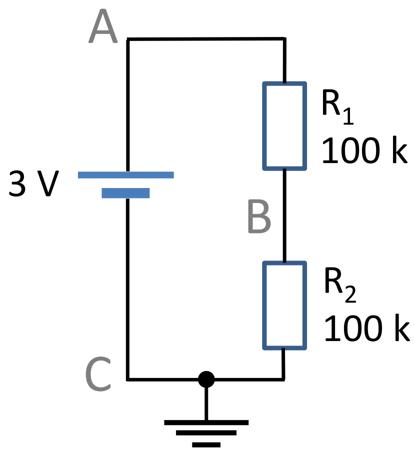


Figure 8.1 A simple circuit to analyse: the potential divider

This circuit has three nodes (labelled as A, B, and C in the circuit diagram above), and three components. Solving this circuit implies working out the voltage at each of these three nodes, and the currents through each of the three components.

We don't need any special techniques to work these out, we could just note that:

- Node C must be at zero volts, since it is connected to ground
- Node A must be at three volts, since the voltage source ensures that it is three volts above node C (which is at zero volts, see above)
- The current must be the same everywhere, since all three components are in series
- The total resistance is 200k, and there is three volts across this 200k, so the current in the circuit must be 3 / 200k = 15 µA
- The voltage at node B (which I'll write as V_B) is then given by Ohm's law applied to R2: V_B = 15 µA x 100 k = 1.5 V.

That's it. We now know the voltages and currents at every point in the circuit.

While simple circuits can be analysed this way, for more complex circuits, or for programming a computer to solve circuits, a more formal algorithm is required. There are two common algorithms:

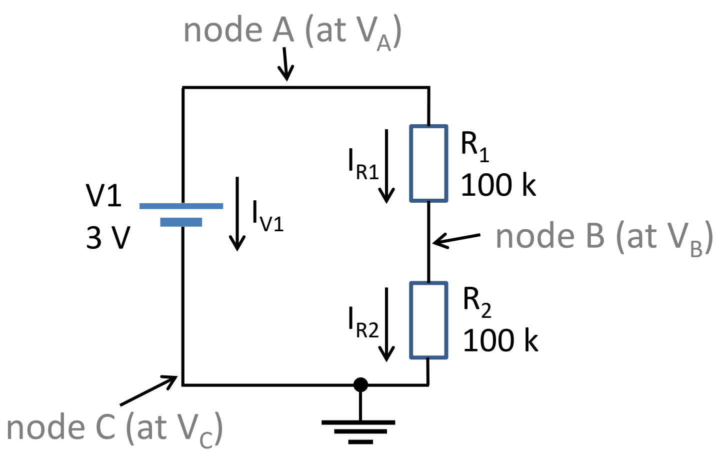
nodal analysis and *mesh analysis*. Nodal analysis is based on Kirchhoff's current law and Ohm's law, mesh analysis is based on Kirchhoff's voltage law and Ohm's law. Both can be used to analyse any given circuit, but since nodal analysis is almost always easier to apply, I'll only describe this algorithm here.

8.2 Nodal analysis

The basic[1] idea of nodal analysis is:

- Work out where all the nodes are in the circuit (a node is a set of component terminals connected together with wires so that they are all at the same voltage)
- Assign a variable name to the voltage at every node
- Assign a variable name to the current through every component
- For each node, write either:
    - If the node is connected to a fixed voltage: an equation defining the voltage[2]
    - Otherwise: an equation applying Kirchhoff's current law to the node
- For each component, write either:
    - For voltage sources: an equation defining the voltage across it
    - For current sources: an equation defining the current through it
    - For resistors: apply Ohm's law to produce an equation relating the current through it to the voltage across it
    - For any other component: an equation relating the voltage across the component to the current flowing through it
- Solve the resultant series of simultaneous equations

For example, applying this algorithm to the potential divider circuit above, we'll first need to identify all the nodes and then add variable names to the voltages and currents through each component:



[1] This is only the basic idea, and only works for circuits which have at least one point connected to ground (or another fixed known voltage) and only contains components with two terminals. We'll look at how to extend the ideas to cover all circuits later.

[2] If you read some other descriptions of nodal analysis, you might find this step referred to as a *supernode*. Personally I think that concept confuses things for those new to circuit analysis so I'm not going to use it here; I prefer to consider voltage sources this way.

Figure 8.2 Potential divider circuit with all voltages and currents marked

Now to write down the equations: first, note that node C is connected to a fixed voltage (in this case ground), so the equation we write for that node is just to define what the voltage here must be:

$$V_C = 0 \tag{8.1}$$

Nodes A and B are not attached to a defined voltage level, so apply Kirchhoff's current law to these nodes[3]:

$$\text{At node A} \Rightarrow \qquad -I_{V1} - I_{R1} = 0 \tag{8.2}$$

$$\text{At node B} \Rightarrow \qquad I_{R1} - I_{R2} = 0 \tag{8.3}$$

Now for the components: for the voltage source, write an equation relating the voltage at each terminal to the value of the voltage source:

$$V_A - V_C = 3 \tag{8.4}$$

and for the resistors, write equations applying Ohm's law to each one:

$$\text{For } R_1 \Rightarrow \qquad V_A - V_B = I_{R1} \times 100\text{k} \tag{8.5}$$

$$\text{For } R_2 \Rightarrow \qquad V_B - V_C = I_{R2} \times 100\text{k} \tag{8.6}$$

That gives us six equations (equations (8.1) to (8.6)) in six unknowns (V_A, V_B, V_C, I_{V1}, I_{R1} and I_{R2}). Solving these by hand is a bit tiresome, but fortunately, there are good, robust ways of solving sets of linear simultaneous equations like these, which can be readily coded into simulation programmes[4].

8.2.1 If you don't have a computer…

While a computer would at this stage just go through the mechanical steps of whatever algorithm it is using to solve these simultaneous equations, a human being can be a bit more intuitive and intelligent when approaching the solution.

For example, in this case we could note that dividing equation (8.5) by equation (8.6) gives:

$$\frac{V_A - V_B}{V_B - V_C} = \frac{I_{R1}}{I_{R2}} \tag{8.7}$$

and then note from equation (8.3) that:

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[3] The most common thing that goes wrong when applying this algorithm is to get the signs of the currents confused. Be careful: it's a good idea to draw arrows on the diagram and remember that current flowing into a node is positive, current flowing away is negative, and the sum of all the currents must be zero.

[4] Provided the circuit is linear, the conventional way is to use matrix notation to represent these equations and use matrix analysis methods to solve them. You will learn about these techniques in the mathematics modules; for the moment it's enough to know that they exist.

$$I_{R1} = I_{R2} \tag{8.8}$$

so:

$$\begin{aligned} \frac{V_A - V_B}{V_B - V_C} &= 1 \\ V_A - V_B &= V_B - V_C \\ 2V_B &= V_A + V_C \end{aligned} \tag{8.9}$$

Then looking at the circuit, it's clear that $V_C = 0$ and $V_A = 3$, so:

$$V_B = \frac{V_A + V_C}{2} = \frac{3+0}{2} = 1.5\text{ V} \tag{8.10}$$

Now that the voltages at all the nodes are known, the currents can be determined from applying Ohm's law to each resistor.

This is much easier than the general problem of solving six simultaneous equations in six unknowns; but it requires some human intelligence to spot the quick way through. There's no substitute for experience here, so it's useful to get a lot of practice.

8.2.2 If none of the nodes is connected to a known voltage…

There is one common case where the simple application of the algorithm described above fails: when no node in the circuit is at a known, defined voltage. For example, suppose the potential divider was drawn like this:

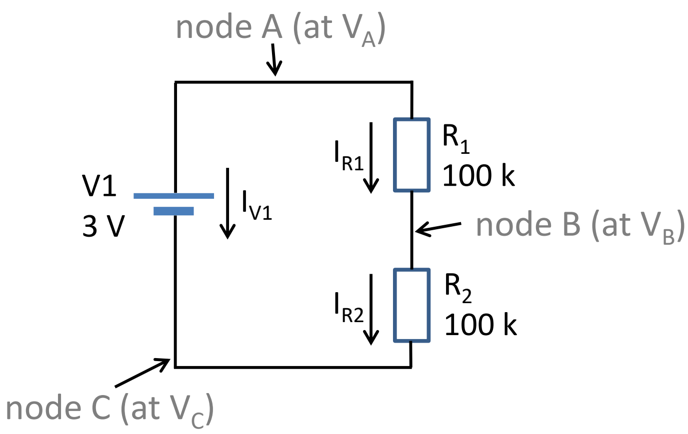

Figure 8.3 A simple circuit with no fixed ground or other known voltage

without node C being defined to be at zero volts. Working through the algorithm above would lead to the six equations:

$$-I_{V1} - I_{R1} = 0 \qquad I_{R1} - I_{R2} = 0 \qquad I_{R2} + I_{V1} = 0 \tag{8.11}$$

$$V_A - V_C = 3 \qquad V_A - V_B = I_{R1} \times 100\text{k} \qquad V_B - V_C = I_{R2} \times 100\text{k} \tag{8.12}$$

and if you try to solve these, you'll find that you can't. The problem is the first three equations (equation (8.11). These three equations are not independent: you can derive any of them from the other two[5]. To solve for six unknowns, you need six independent equations, all of which give new information; and here there are only five (as the top three equations only have two pieces of information in them).

If you think about it, you might agree that this is perfectly reasonable: if no node in the circuit is at a known voltage, then it's impossible to work out what voltage any other node can be at: voltage sources and Ohm's law are only concerned with the voltage difference between two nodes, not with the absolute value of the voltage anywhere.

The solution is simple: if there is no defined ground in the circuit, then you're free to choose where ground is. Choose any node[6], define it to be at zero volts, and then apply the algorithm[7]. From the solution, you can determine the relative voltage at two nodes (by subtracting the voltage at each node), but you can't say anything about the absolute voltage at any point in the circuit, and that's the best it's possible to do.

In this case, this initial step would just transform the circuit in Figure 8.3 back into that of Figure 8.2.

8.3 A (slightly) more complex example

In the chapter on linearity and superposition, we considered this circuit:

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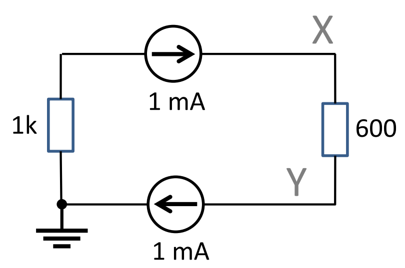
[5] For example, add any two of the equations together, then multiply the result by minus one: the result will be the other equation.

[6] Any node would work, but to make the maths easier it's a good idea to choose the node with the largest number of connections to other components. If there isn't a node with a larger number than any other (as is the case here) then it really doesn't matter which node you pick, but I would suggest choosing the one at the negative terminal of the voltage source, so that most voltages in the circuit end up being positive.

[7] For the sake of completeness I should perhaps point out that this doesn't solve the problem in all possible circuits, but it does solve the problem in all circuits of interest for this module, so it'll do for now. One example of a circuit for which this algorithm fails would be:



where the nodal analysis technique can't work out the voltages at nodes X and Y (although it can work out that the voltage difference between them must be 0.6 V).

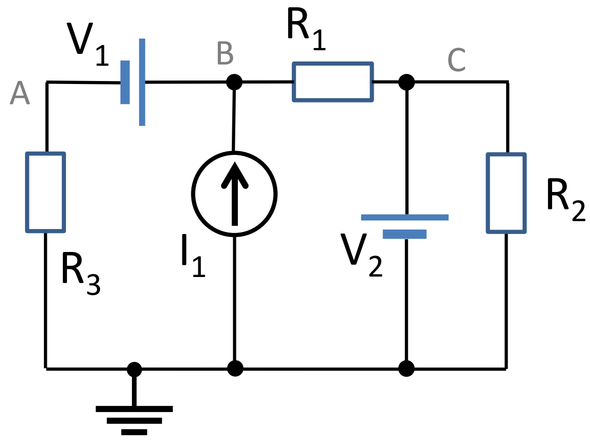


Figure 8.4 A circuit employing voltage and current sources

and showed how it could be solved using the principle of superposition. The same circuit can of course be analysed more formally through nodal analysis. This circuit does have one node attached to a fixed, known, absolute voltage (ground), so we don't need to define a ground, and we can immediately start by labelling all the currents and voltages:

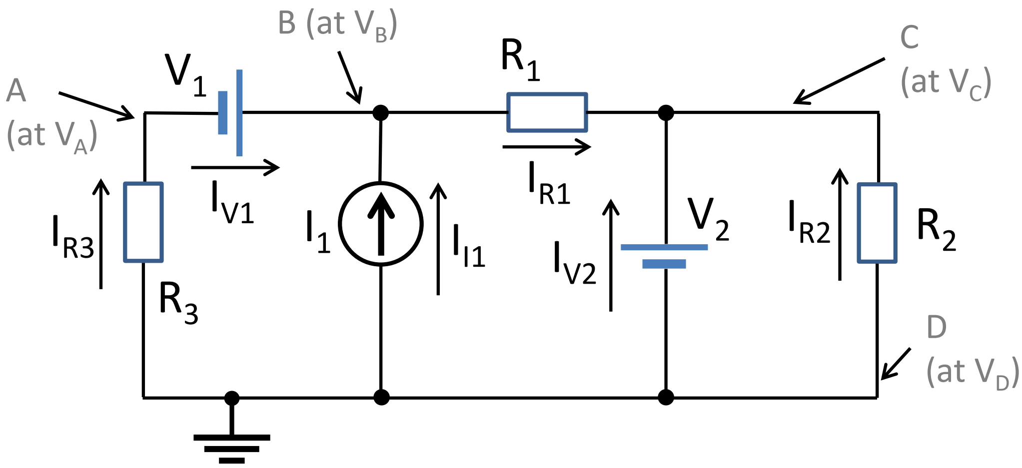


Figure 8.5 More complex circuit with all nodes, voltages and currents labelled

Then work through writing the equations. Firstly, node D is at a fixed, defined voltage, so we can immediately write:

$$V_D = 0 \tag{8.13}$$

Then, apply Kirchhoff's current law to the other three nodes, taking care to get the directions of the currents correct:

$$I_{R3} = I_{V1} \tag{8.14}$$

$$I_{V1} + I_{I1} = I_{R1} \tag{8.15}$$

$$I_{R1} + I_{V2} = I_{R2} \tag{8.16}$$

Then for the voltage sources:

$$V_B - V_A = V_1 \tag{8.17}$$

$$V_C - V_D = V_2 \tag{8.18}$$

then the current source:

$$I_{I1} = I_1 \tag{8.19}$$

and finally apply Ohm's law to the three resistors, again taking care to ensure that positive current flows from the higher potential to the lower potential:

$$V_D - V_A = I_{R3} \times R_3 \tag{8.20}$$

$$V_B - V_C = I_{R1} \times R_1 \tag{8.21}$$

$$V_D - V_C = I_{R2} \times R_2 \tag{8.22}$$

That's ten independent equations in ten unknowns (six currents and four voltages).

Of course, any sensible person wouldn't bother to write $V_D = 0$, would automatically write V_C just as V_2, and would note that $I_{R3} = I_{V1}$ and therefore there's no point in giving them different symbols. That's three unknowns dealt with already. However a computer would just work through solving the equations above.

The trick to manual circuit analysis is to spot these sorts of ways to minimise the number of unknowns and equations you have to solve.

8.4 Alternative techniques and short-cuts

We've already seen one short cut to working out the circuit in Figure 8.5: using superposition to determine the value of I_{R1}, the current through the resistor R_1 (see the chapter on linearity and superposition). There are a few other techniques worth knowing about that can speed things up as well.

8.4.1 Using Thévenin's theorem

Suppose we have the same problem: to work out the current though R_1 in the circuit in Figure 8.5. We could approach this problem using Thévenin's theorem. First, separate the circuit into three: the components to the left of R_1; R_1 itself; and the components to the right of R_1, as shown in Figure 8.6.

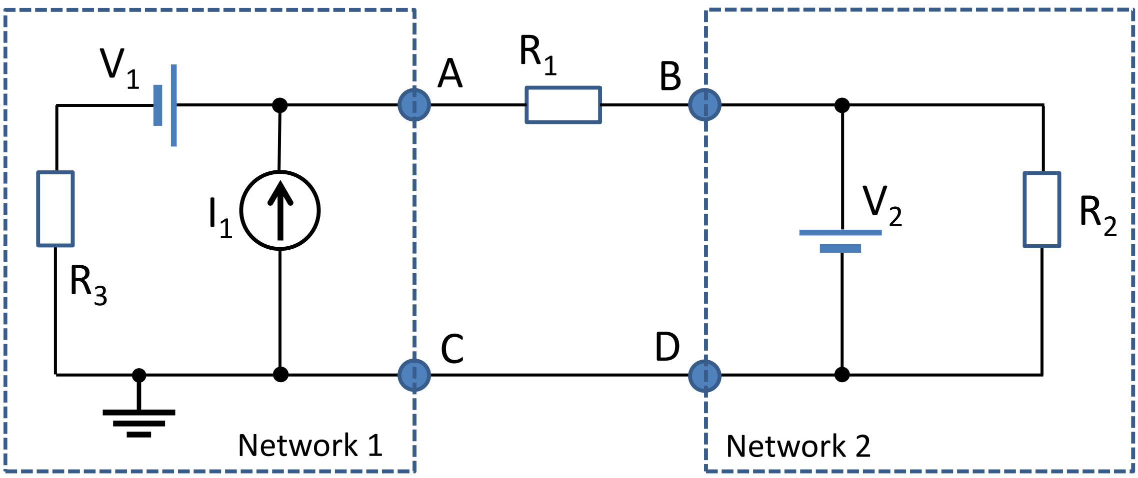


Figure 8.6 Decomposing the circuit into three networks and a resistor

The next stage in this technique is to work out the Thévenin equivalents for the two networks shown as "Network 1" and "Network 2". You might be able to see that the open circuit voltage of network 1 is:

$$V_{OC\_1} = I_1 R_3 + V_1 \tag{8.23}$$

since it must be the voltage across R_3 with a current of I_1 through it, plus the voltage across V_1. Similarly, using the "set all the sources to zero" trick, the Thévenin-equivalent resistor for this network is:

$$V_{TH\_1} = R_3 \tag{8.24}$$

On the other side, the open-circuit voltage is obviously just:

$$V_{OC\_2} = V_2 \tag{8.25}$$

and again, the "set all sources to zero" trick reveals that the Thévenin-equivalent resistor in this case is:

$$V_{TH\_2} = 0 \tag{8.26}$$

since the voltage source would just turn into a wire when set to zero, making the resistance between points B and D zero. Replacing these two networks with their Thévenin equivalents then produces the circuit shown in Figure 8.7, which from the point of view of R_1, must work just the same:

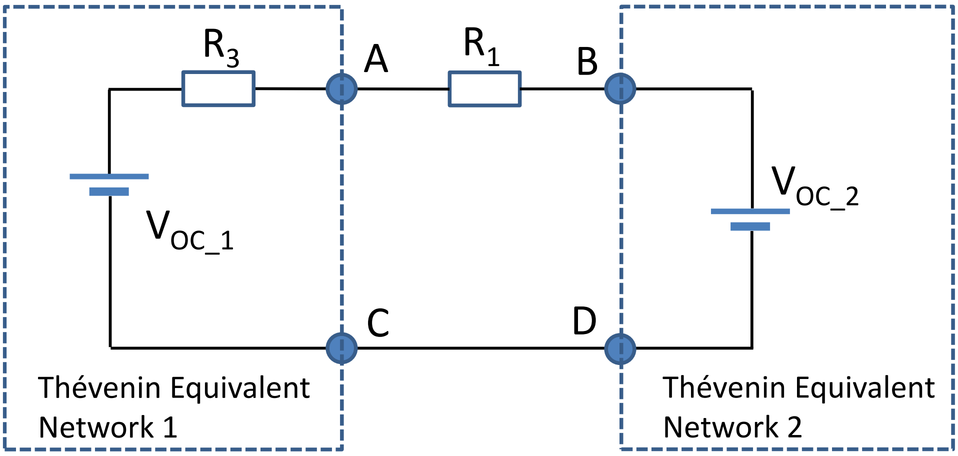


Figure 8.7 Simplified circuit using Thévenin equivalent networks

This is much easier: it's now very easy to show that the current through R_1 must be:

$$I_{R1} = \frac{V_{OC\_1} - V_{OC\_2}}{R_3 + R_1} = \frac{I_1 R_3 + V_1 - V_2}{R_3 + R_1} \tag{8.27}$$

and this is perhaps the simplest way of deriving the result found so far.

8.4.2 Source transformation

A related technique that is useful in some circuits is known as *source transformation*. Recall[8] that you can represent any linear two-terminal network in terms of either a Thévenin or Norton equivalent circuit: the Thévenin equivalent having a voltage source and series resistor; the Norton equivalent having a current source and parallel resistor. Further, the resistors in both cases are equal, and the Thévenin equivalent voltage source V_{TH} is related to the Norton equivalent current source I_N by:

$$V_{TH} = I_N \times R_{TH} \tag{8.28}$$

which could equally well be written as:

$$V_{TH} = I_N \times R_N \tag{8.29}$$

since the Thévenin equivalent resistance R_{TH} is equal to the Norton equivalent resistance R_N. Graphically, this could be represented as follows:

[8] For more details see the chapter on Thévenin and Norton equivalent circuits.

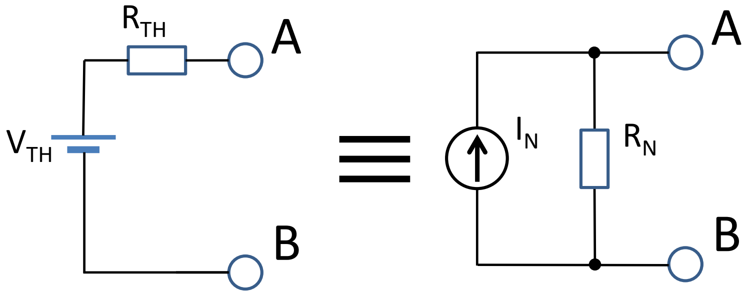


Figure 8.8 Two equivalent circuits: $R_{TH} = R_N$, $V_{TH} = I_N \times R_N$

The method of source transformation consists of constantly switching back and forth between the Thévenin and Norton representations, including one additional component in the equivalent circuit at each stage. For example: consider the same circuit again:

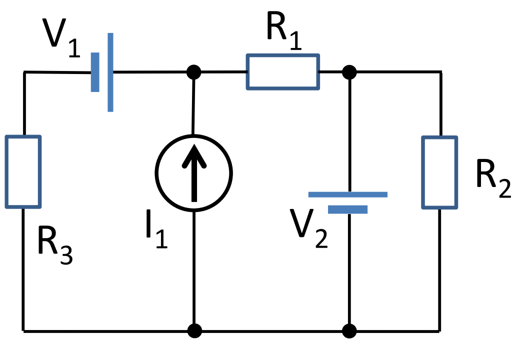


Figure 8.9 Circuit for analysis using source transformation

I'll start on the left-hand side, and the two components R_3 and V_1. Setting two points in the circuit (marked P and S as shown in Figure 8.10 below), the components on the left-hand side of P and S are already in the form of a Thévenin equivalent network (a voltage source and a resistor in series).

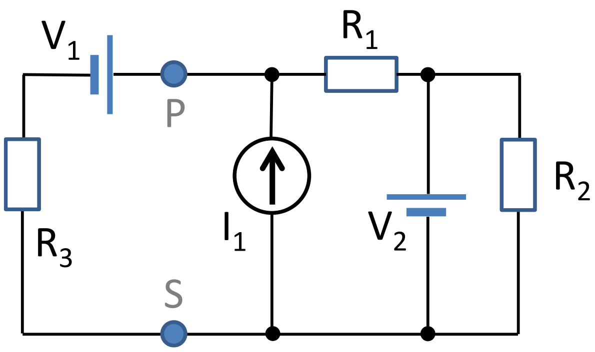


Figure 8.10 Source transformation example stage one: setting the scene

Once we have the Thévenin equivalent network, we can transform this into a Norton equivalent network which contains a current source I_2 (of magnitude V_1 / R_3 in this case) in parallel with a resistor R_4 (which has the same resistance as R_3)[9]. The result is the circuit shown in Figure 8.11.

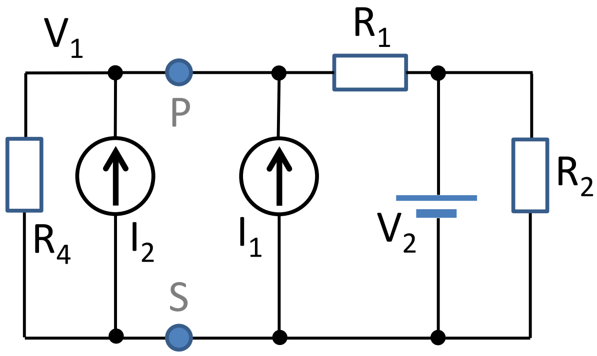


Figure 8.11 Source transformation example stage two: transforming the first voltage source

Note here that we have two current sources in parallel, and two current sources in parallel can be easily combined by replacing them with a single current source which has a value of the sum of the two current sources (after all, in both cases, a current of $I_1 + I_2$ is required to flow up through the current sources from S to P). So first move the second current source to the other side of the points P and S:

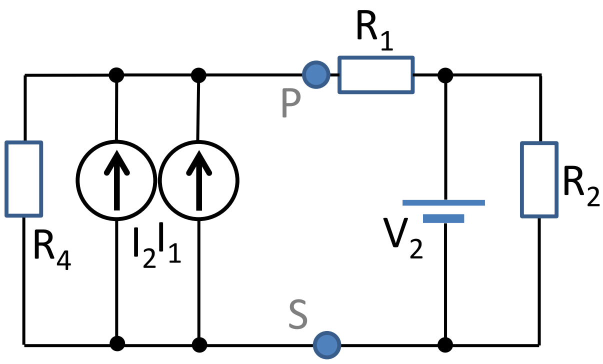


Figure 8.12 Source transformation example stage three: combining current sources

and then combine them into one larger current source:

[9] It's important to note that this is not the original resistor R_3. It's another resistor which happens to have the same resistance value as R_3. (That's why I've called it R_4 here.) It has a different voltage across it, and a different current through it than the original resistor R_3.

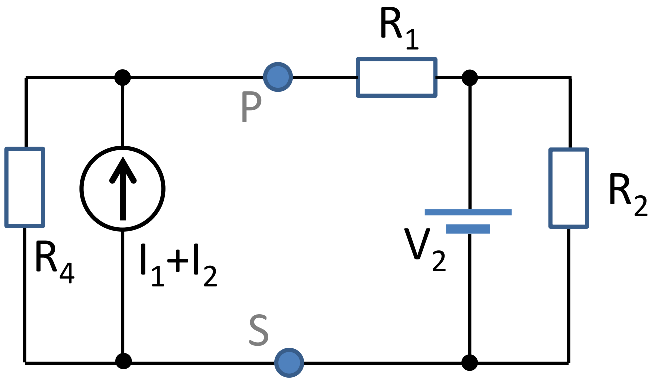


Figure 8.13 Source transformation example stage four: combined current sources

Now we have a Norton equivalent network on the left-hand side of points P and S, and we can easily transform that back into a Thévenin equivalent network:

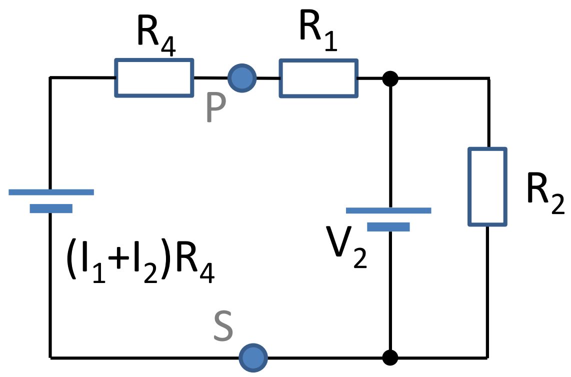


Figure 8.14 Source transformation example stage five: transforming back to a voltage source

The reason for this transform back is that the next component along towards the right-hand side of the circuit is a resistor R_1 in series, and a Thévenin equivalent network would put another resistor in series with this resistor when points P and S are moved to the right again:

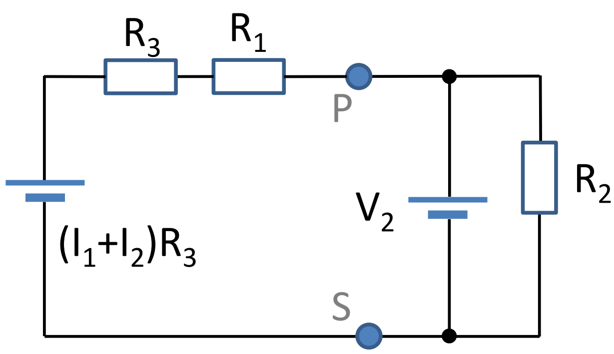


Figure 8.15 Source transformation example stage six: moving P and S again

and resistors in series are easy to combine and replace with a single resistor. However in this case what we want to know is the current through R_1, so there is no need to take this step[10].

In general, for a more complex circuit, we could follow a similar set of steps from the right-hand end of the circuit until R_1, but in this case there is no need to do that: we already know what the voltage on the right-hand side of R_1 is, since it is defined by V_2.

Since we know that $I_2 = V_1 / R_3$ it's immediately clear that the voltage on the left-hand side of the resistors R_1 and R_4 is $(V_1 / R_3 + I_1)$ above the bottom node in the circuit, and on the right-hand side it is V_2 above the bottom node, so the voltage across the resistors must be:

$$V_{across} = \frac{V_1}{R_3} + I_1 - V_2 \tag{8.30}$$

which means the current through the resistor R_1 (and R_4) must be:

$$V_{across} = \frac{I_2 + I_1 - V_2}{R_1 + R_4} = \frac{\frac{V_1}{R_3} + I_1 - V_2}{R_1 + R_3} \tag{8.31}$$

and that's perhaps the easiest way to get to the result yet.

8.4.3 Source transformation in general

Source transformation is most useful in analysing circuits which are in the form:

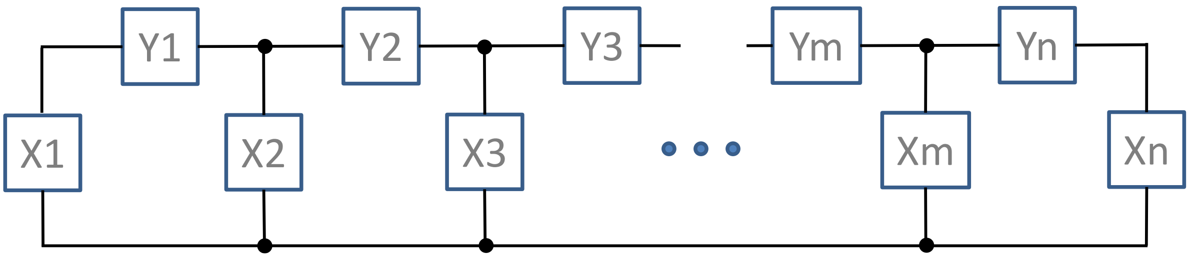


Figure 8.16 Generalised ladder circuit

where X1, Y1, X2 etc. can be independent voltage sources, independent current sources or resistors.

This sequence of parallel and series components is sometimes called a *ladder* circuit. To determine the voltage at any point, or current through any component in a ladder circuit, you just start at each end, and work through, absorbing one component at a time into the equivalent circuit, until you get to the node or component of interest.

The rules can be formalised:

- If the next component is a series resistance (e.g. Y2 or Y3), then transform to a Thévenin equivalent network, and add the new resistance to the Thévenin resistance.

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[10] In fact we can't really take it, since if we're trying to find out the current through R_1 we need to keep R_1 in the circuit.

- If the next component is a series voltage source, then transform to a Thévenin equivalent network and add the new voltage source to the Thévenin voltage source.
- If the next component is a parallel resistance (e.g. X2 or X3), then transform to a Norton equivalent network, and combine the new resistance in parallel with the Norton resistance.
- If the next component is a parallel current source, then transform to a Norton equivalent network, and add the current source to the Norton current source.

You might be wondering what to do if the next component is a parallel voltage source, or a series current source (e.g. if X2 was a voltage source, or Y2 was a current source). The answer: if this happens, then you've been wasting your time. For example, if X2 is a voltage source, then it doesn't matter what X1 and Y1 are: it makes no difference to the voltage above X2, and therefore no difference to anything in the circuit to the right of X2. You don't need to consider these components at all if you're only interested in the voltages or currents to the right of this point. Similarly if Y2 is a current source, then X1, Y1 and X2 would have no effect on the values of voltages or currents to the right of Y2. As far as the right-hand side of the circuit is concerned, the circuit would behave like this:

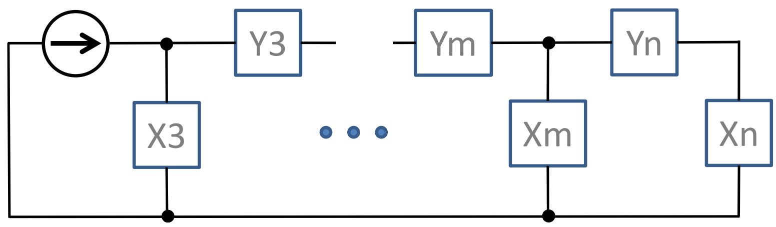

Figure 8.17 Ladder circuit with Y2 as a current source

Another way to think about this is to realise that the equivalent circuit of any network of the form:

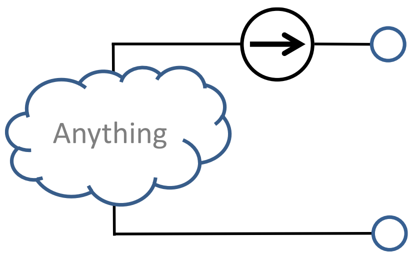

Figure 8.18 General circuit with a Norton equivalent network of a single current source

is just a single current source, since the short-circuit current is the current of the current source, and the Norton equivalent resistance is infinite (set the current source to zero, and no current would flow between the two terminals, so the equivalent network resistance is infinite), and the equivalent circuit of any network of the form:

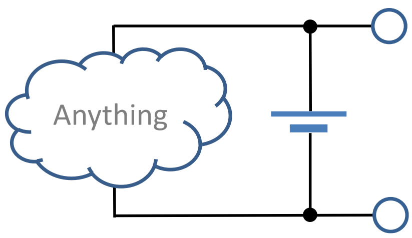

Figure 8.19 General circuit with a Norton equivalent network of a single current source

is just a single voltage source, since the Thévenin equivalent resistance is zero (set the voltage source to zero, and there would be a short circuit between the two terminals).

8.4.4 Another example of source transformation

I'll do another example, this time with some numbers. Consider the following circuit, in which you want to know the voltage at the node marked 'N', which I'll refer to as V_N.

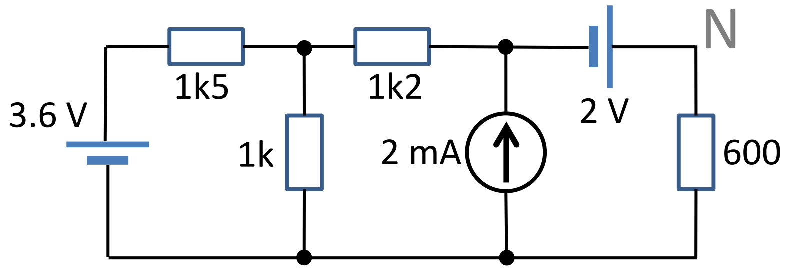

Figure 8.20 Second source transformation example stage one

The first step is to consider two terminals for the equivalent network, here I'll call them 'A' and 'B':

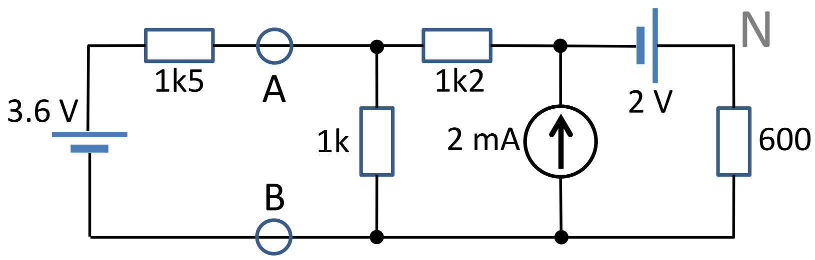

Figure 8.21 Second source transformation example stage two

The circuit to the left of these terminals is in a Thévenin equivalent form (a voltage source and a series resistor). Since the next component along is a parallel resistor to ground, we need to transform the network to the left of 'A' and 'B' from Thévenin to Norton equivalent form:

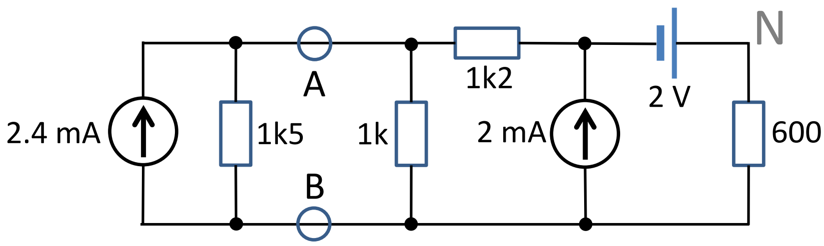

Figure 8.22 Second source transformation example stage three

where the Norton equivalent current is 3.6 V / 1k5 = 2.4 mA, and the equivalent Norton resistor is the same as for the Thévenin equivalent network. At this point we can move the points 'A' and 'B' one component to the right:

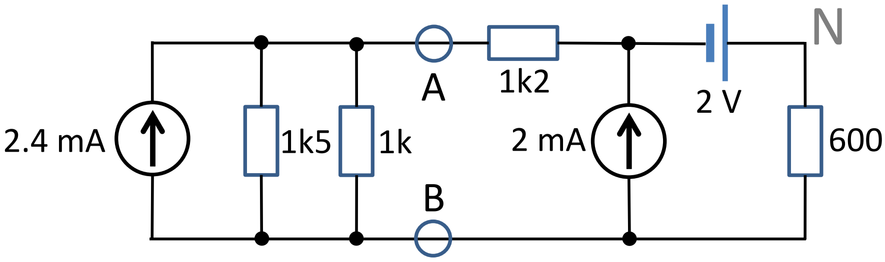

Figure 8.23 Second source transformation example stage four

and then combine the 1k5 and 1k resistors (which are in parallel) to form a single resistor with a value of 1k5 * 1k / (1k5 + 1k) = 600 ohms:

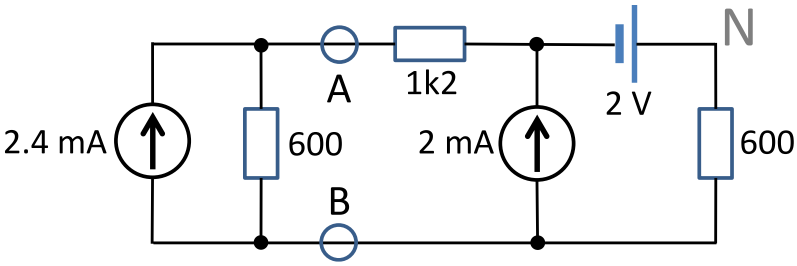

Figure 8.24 Second source transformation example stage five

The next component along is a series resistor, so we convert the network to the left of ‘A’ and ‘B’ back to a Thévenin equivalent, where the Thévenin equivalent voltage (the open-circuit voltage) is given by 2.4m * 600 = 1.44 V, and the resistor is the same value as in the Norton equivalent network:

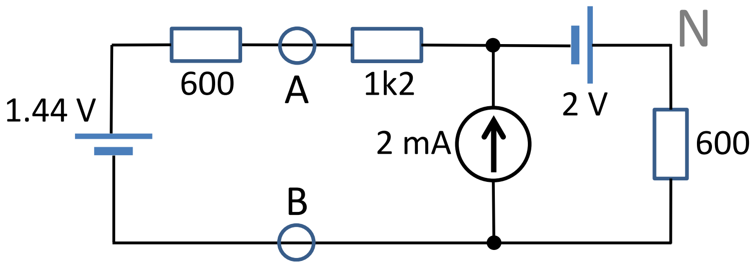

Figure 8.25 Second source transformation example stage six

Now we can move the points ‘A’ and ‘B’ along past the 1k2 resistor, and the 600 and 1k2 resistors (which are in series) can be combined into a single resistor of value 600 + 1k2 = 1k8:

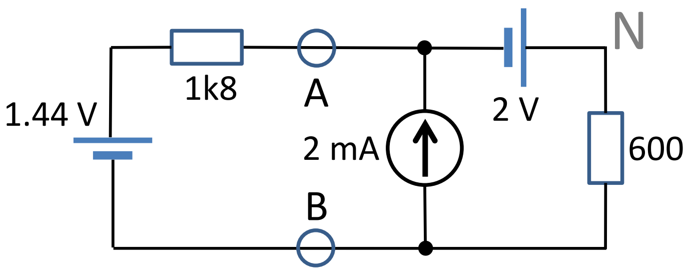

Figure 8.26 Second source transformation example stage seven

The next component is a parallel current source, so we need to convert the network to the left of 'A' and 'B' into a Norton equivalent circuit, with a current source of 1.44 V / 1k8 = 0.8 mA and an equivalent resistor of value 1k8:

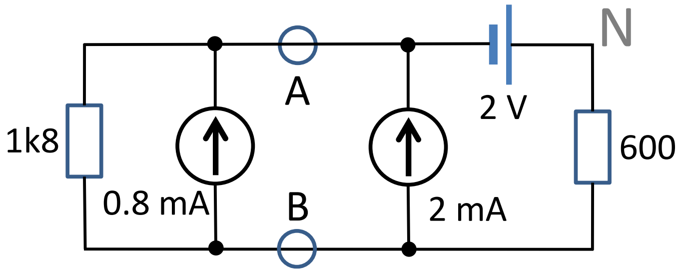

Figure 8.27 Second source transformation example stage eight

Now we can move the points 'A' and 'B' to the right again, and combine the two current sources into a single current source of value 0.8 mA + 2 mA = 2.8 mA:

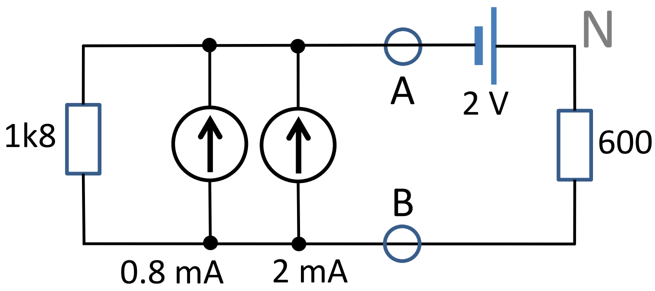

Figure 8.28 Second source transformation example stage nine

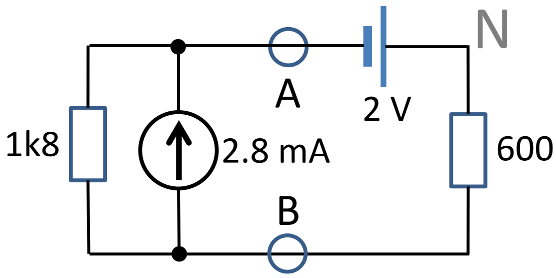

Figure 8.29 Second source transformation example stage ten

The next component is a series voltage source, so we convert the network to the left of 'A' and 'B' to a Thévenin equivalent network, with a voltage source of 2.8 mA * 1k8 = 5.04 V, and a 1k8 resistor:

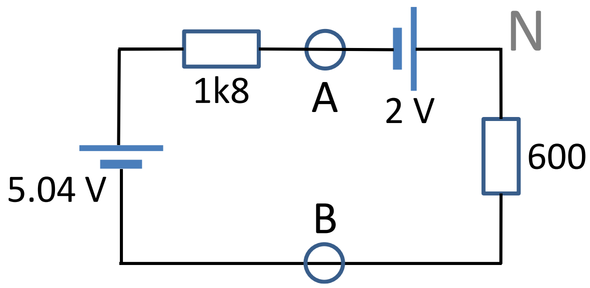

Figure 8.30 Second source transformation example stage eleven

and moving the points ‘A’ and ‘B’ to the other side of the voltage source V2 gives:

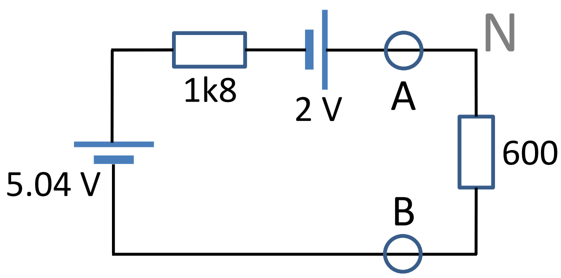

Figure 8.31 Second source transformation example stage twelve

You can always change the order of series components, so this is equivalent to:

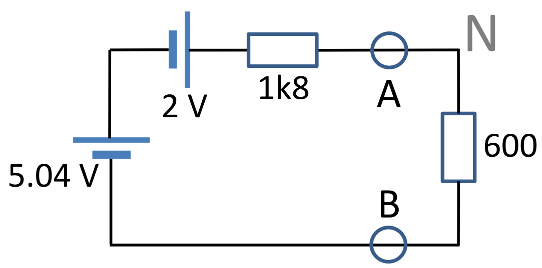

Figure 8.32 Second source transformation example stage thirteen

and we can replace these two voltage sources with a single voltage source of 5.04 + 2 = 7.04 V:

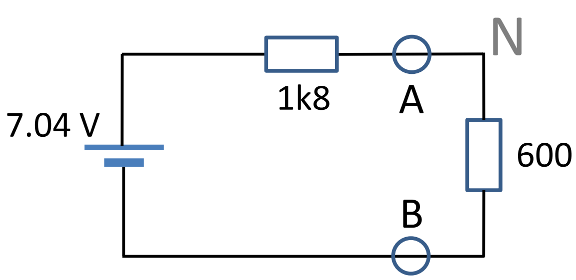

Figure 8.33 Second source transformation example stage fourteen

At this point it’s just a potential divider, so we can immediately write for the voltage at N:

$$V_N = 7.04 \frac{600}{1800 + 600} = 1.76 \text{ V} \tag{8.32}$$

While time-consuming, each individual step in this process is quite simple, and as a result this approach can be considerably easier than trying to solve a large set of simultaneous equations.

8.5 Dependent sources

Some circuits have voltage and current sources which don't have a fixed value; their value is dependent on the value of the voltages or currents elsewhere in the circuit. These are known as *dependent sources*. They present no difficulty for the nodal analysis algorithm, and no problems with superposition (provided you remember to only set the independent sources to zero in turn). However, they can present difficulties for the equivalent circuit-based approaches: see the chapter on Thévenin and Norton's theorems for more details about the problems in calculating the equivalent circuit in these cases, so source transformation techniques are best avoided in these cases.

For an example of a circuit analysis including a dependent source, consider the circuit shown in Figure 8.34:

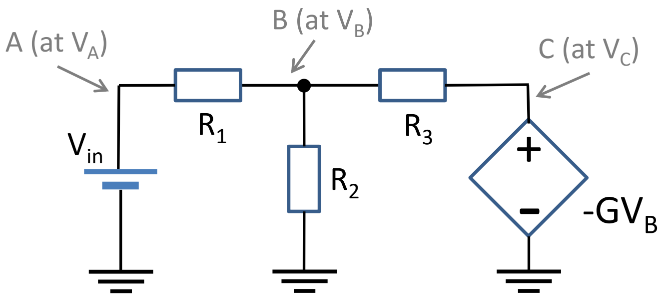

Figure 8.34 Circuit with a dependent voltage source

where the voltage source on the right of the diagram (in the diagonal square box) is a dependent voltage source, which takes a value given by –G times the value of the voltage at node B.

The formal nodal analysis process follows through exactly as before: first assign names for all the nodes (as shown above) and then all the currents (and here I'll refer to the current through R_1 as i_{R1}, and so on, as shown below):

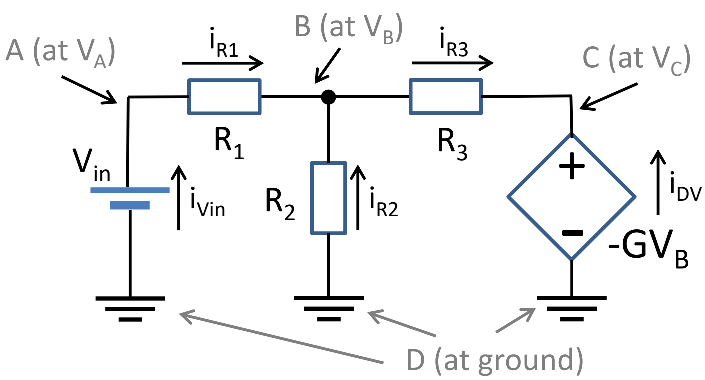

Figure 8.35 Circuit with a dependent voltage source and labelled currents

The equations for the nodes are now:

$$\begin{array}{cc} \text{node A} & i_{Vin} = i_{R1} \\ \text{node B} & i_{R1} + i_{R2} = i_{R3} \\ \text{node C} & i_{R3} + i_{DV} = 0 \\ \text{node D} & V_D = 0 \end{array} \tag{8.33}$$

and the equations for the components:

$$\begin{array}{cc} \text{for } V_{\text{in}} & V_A - V_D = V_{in} \\ \text{for } R_1 & V_A - V_B = i_{R1} R_1 \\ \text{for } R_2 & V_B - V_D = -i_{R2} R_2 \\ \text{for } R_3 & V_B - V_C = i_{R3} R_3 \\ \text{for the dependent source} & V_C = -G V_B \end{array} \tag{8.34}$$

That's nine equations in nine unknowns (the four voltages V_A, V_B, V_C, V_D and the five currents).

Experienced humans probably wouldn't even write out all of these equations; they would notice that V_D is at zero, V_A is at V_{in} and that $i_{Vin} = i_{R1}$ and $i_{R3} = -i_{DV}$ immediately, and use Ohm's laws to substitute for the currents before putting pen to paper, and then just write:

$$\frac{V_{in} - V_B}{R_1} - \frac{V_B}{R_2} = \frac{V_B - V_C}{R_3} \tag{8.35}$$

from applying Kirchhoff's current law to node B, and:

$$V_C = -G V_B \tag{8.36}$$

to represent what the dependent voltage source is doing. This gives just two equations in two unknowns (V_B and V_C), and that's much easier to solve. Eliminating V_B quickly gives:

$$V_C = \frac{-V_{in}}{R_1\left(\frac{2}{GR_1} + \frac{1}{GR_3} + \frac{1}{R_3}\right)} \tag{8.37}$$

(You might find it interesting to note that when G is very large, this can be approximated as:

$$V_C = -V_{in}\frac{R_3}{R_1} \tag{8.38}$$

which is not a function of G at all. This an example of negative feedback: see the chapter on op-amps for more information about this topic.)

The skill in manual circuit analysis comes from spotting how to minimise the number of variables and number of equations that have to be solved, and like most skills, this one comes with practice.

8.6 Summary: the most important things to know

- Circuit analysis can always be done using the method of nodal analysis. This involves:
    - Applying Kirchhoff's current law to nodes in the circuit not at a fixed voltage
    - Specifying the relationship between current and voltage difference (e.g. Ohm's law) for all components
    - Solving the resultant simultaneous equations
    - This can be complex and take a long time.
- A useful short-cut is to determine Thévenin or Norton equivalents for linear sections of the circuit with two terminals.