7 A Short Introduction to Circuit Loading

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Prerequisite knowledge required: Ohm's Law and Kirchhoff's Laws, Thévenin's Theorem

7.1 Introduction

You might have experienced the problem yourself: two microphones both work perfectly fine with one system, but in a different system one gives a much higher signal level than the other. Or perhaps two pairs of headphones both work fine with one MP3 player, but only one of them works with a different player. What's going on?

Most likely, the problem is to do with circuit loading: the fact that the behaviour of a circuit is dependent on what is connected to its input and output. This chapter will introduce the idea of an equivalent input and output network and show how they can be used to predict the behaviour of any circuit with any input and output, so unexpected problems are less likely to occur.

7.2 Revision of Thévenin's theorem

At this point a quick reminder of Thévenin's theorem might be useful. Thévenin's theorem states that any two-terminal linear network has an equivalent network which consists of a voltage source and a resistor in series[1].

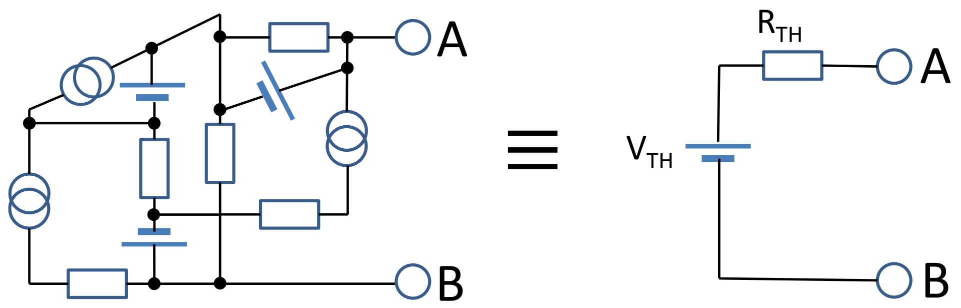

Figure 7.1 Illustration of Thévenin's theorem: the two networks behave in the same way for any other networks connected to the two terminals

This works both for sources (for example microphones), and for sinks[2] (for example the inputs to a mixing desk or amplifier). Both can be represented by an equivalent Thévenin network (although in

[1] If you're reading this after you've read about ac circuit analysis, then please note this is the original version of the theorem, which was derived for dc sources and resistors only. The theory can be generalised to ac sources and impedances.

[2] In this context, a sink is the opposite of a source. Signals start from sources, and end at sinks. It's important to note that signals are not the same thing as currents: currents don't start anywhere; they just go round in circuits.

the case of the sink, the Thévenin voltage V_{TH} is ideally zero, so the input looks just like a resistance[3], usually called the *input resistance*).

With an understanding of Thévenin equivalent circuits, we can return to the original question: what's going on with those microphones and headphones?

7.3 Microphones and input resistance

First, consider the microphones. The output of the microphone has two terminals (a signal and a ground return), and it's a linear circuit (since a doubling of the sound pressure causes a doubling of the electrical output signal) so it has a Thévenin equivalent network. The input to the amplifier to which it's connected will also have a Thévenin equivalent network, and will almost certainly look like a resistor to ground. So the circuit made by connecting the microphone to the amplifier would look like this:

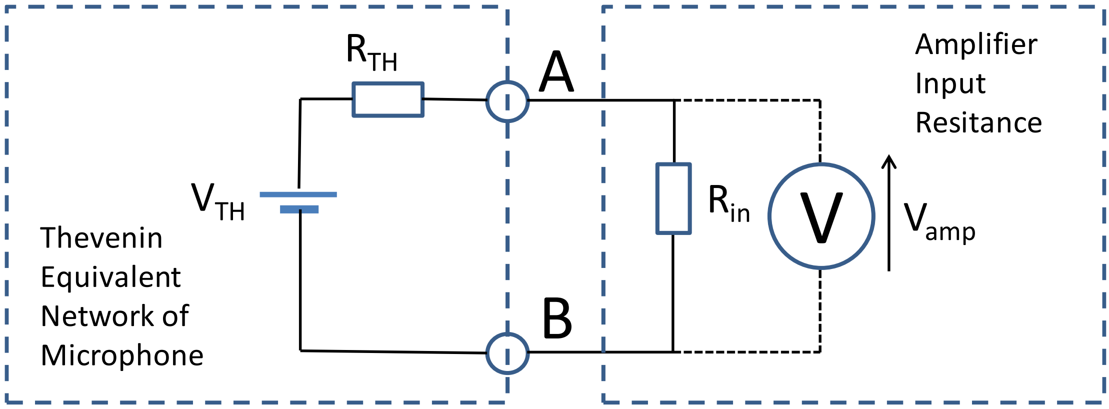

Figure 7.2 - Equivalent circuit of a microphone connected to an amplifier input

Looking at this circuit, it's just a potential divider, so we can immediately write:

$$V_{amp} = V_{TH} \frac{R_{in}}{R_{TH} + R_{in}} \tag{7.1}$$

If the input resistance to the amplifier R_{in} is much greater than the Thévenin equivalent resistance of the microphone R_{TH}, then we could write:

$$\begin{aligned} & R_{TH} + R_{in} \approx R_{in} \\ & V_{amp} = V_{TH} \frac{R_{in}}{R_{TH} + R_{in}} \approx V_{TH} \frac{R_{in}}{R_{in}} = V_{TH} \end{aligned} \tag{7.2}$$

and in this case it doesn't really matter exactly what the Thévenin equivalent resistance of the microphone is, as long as the input resistance of the amplifier is much greater, the voltage received by the amplifier will be the same. With a sufficiently high R_{in}, any microphone will work.

[3] Again I should add a note here that for real microphones, which produce ac signals rather than dc signals, we should be talking about impedances rather than resistances, and that for real devices, the Thévenin equivalent impedance is a function of frequency.

However, if we use an amplifier with a lower input impedance, then any microphone with a large Thévenin resistance will suffer, producing only a fraction of its open-circuit voltage V_{TH} at the input to the amplifier V_{amp}.

7.3.1 Headphones and output resistance

Next, consider the headphones. The output of an amplifier will also have a Thévenin equivalent voltage and resistance. It's effectively the same problem, but this time with the amplifier output feeding the headphones instead of the microphone feeding the amplifier input. Again, we have a potential divider (see Figure 7.3), and the result this time works out to be:

$$V_{headphone} = V_{TH} \frac{R_{headphone}}{R_{TH} + R_{headphone}} \tag{7.3}$$

When the headphones have a resistance much smaller than the output resistance of the amplifier, then voltage at the output of the amplifier will be small:

$$V_{headphone} = V_{TH} \frac{R_{headphone}}{R_{TH} + R_{headphone}} \approx V_{TH} \frac{R_{headphone}}{R_{TH}} \qquad \left(\mathrm{R_{headphone}} \ll \mathrm{R_{TH}}\right) \tag{7.4}$$

whereas with a headphone resistance much larger than the output resistance of the amplifier, the voltage at the output of the amplifier is much larger, and almost equal to V_{TH}:

$$V_{headphone} = V_{TH} \frac{R_{headphone}}{R_{TH} + R_{headphone}} \approx V_{TH} \frac{R_{headphone}}{R_{headphone}} = V_{TH} \qquad \left(\mathrm{R_{headphone}} \gg \mathrm{R_{TH}}\right) \tag{7.5}$$

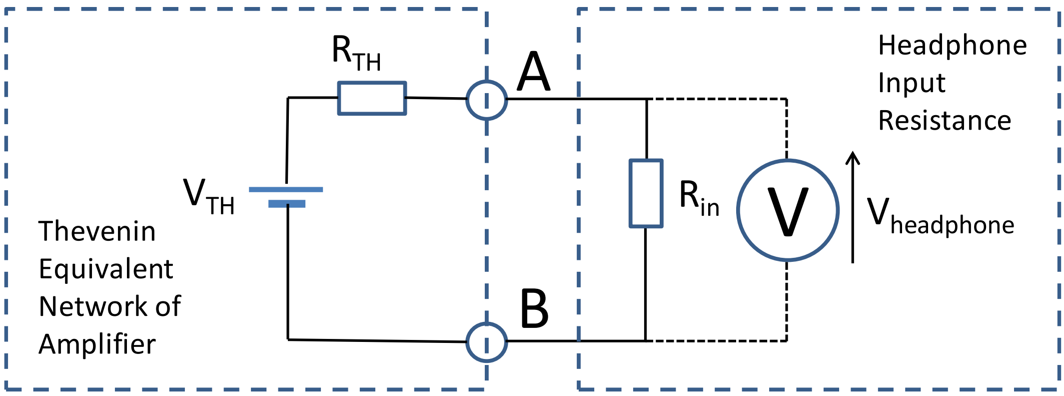

Figure 7.3 - Equivalent circuit of a headphone connected to an amplifier output

Note that if the headphone input resistance is equal to the amplifier output resistance, the voltage across the headphones is half of the open-circuit output voltage from the amplifier.

7.4 Some numeric examples

Suppose we had an amplifier with an open-circuit output voltage (Thévenin voltage) of one volt, and an output resistance (Thévenin resistance) of 100 ohms. Driving headphones with a resistance of 250 ohms, we'd expect the output voltage to be:

$$V_{out} = V_{TH}\frac{R_{load}}{R_{TH}+R_{load}} = 1\times\frac{250}{250+100} = \frac{250}{350} = 714.3\text{ mV} \tag{7.6}$$

which delivers around 2 mW into the load. However the same amplifier driving a speaker with a resistance of 8 ohms would give at its output:

$$V_{out} = V_{TH}\frac{R_{load}}{R_{TH}+R_{load}} = 1\times\frac{8}{8+100} = \frac{8}{108} = 74.1\text{ mV} \tag{7.7}$$

which only delivers a little under 700 µW of useful power into the load.

Another example: suppose a microphone with an output resistance (Thévenin resistance) of 1k was connected to the input of an amplifier with an input resistance of 600 ohms. The voltage at the amplifier input would be:

$$V_{in} = V_{TH}\frac{600}{1000+600} = V_{TH}\frac{600}{1600} = 0.375\,V_{TH} \tag{7.8}$$

so that only around 37.5% of the microphone's open circuit output voltage would be available at this amplifier's input. Whereas with an amplifier with an input resistance of 50k, the received signal would be:

$$V_{in} = V_{TH}\frac{50000}{1000+50000} = V_{TH}\frac{50000}{51000} = 0.98\,V_{TH} \tag{7.9}$$

and 98% of the microphone's open circuit output circuit would be received.

7.5 Power transfer and the maximum power theorem

Above, I calculated not only the voltage at the headphone or speaker output, but also the power delivered into this load. This is because what's really important, in terms of the volume of sound produced, is the amount of power delivered to the headphones.

The power dissipated in a resistance, as you may remember, is given by:

$$P = V\times I = \frac{V^2}{R} = I^2R \tag{7.10}$$

So while having very high resistance headphones would certainly maximise the voltage level at the output of the amplifier, it does not necessarily maximise the power delivered to the load (and hence the energy available to create sound). In the extreme case, with headphones with an infinite resistance, the voltage across the headphones would be at its absolute maximum (the open-circuit voltage from the amplifier's output), but the headphones would make no sound at all, since with no current flowing there is zero power available to be converted into sound.

This leads to a very interesting and important question: if having no resistance across the terminals results in no voltage across the output and therefore no power, and having an infinite resistance results in no current through the output load and therefore no power, what load resistance should we choose to get the maximum amount of power delivered to the load?

This result is known as the *maximum power theorem*. You can work it out quite easily: first, note that the power delivered to the load is:

$$P = \frac{{V_{load}}^2}{R_{load}} \tag{7.11}$$

and that the voltage across the load is given by equation (7.3):

$$V_{load} = V_{TH} \frac{R_{load}}{R_{TH} + R_{load}} \tag{7.12}$$

where V_{TH} and R_{TH} are the open-circuit voltage and output resistance of the amplifier respectively. So:

$$P = \frac{1}{R_{load}} \left(V_{TH} \frac{R_{load}}{R_{TH} + R_{load}} \right)^2 = {V_{TH}}^2 \frac{R_{load}}{\left(R_{TH} + R_{load}\right)^2} \tag{7.13}$$

which when plotted (note the logarithmic scale on the x-axis), gives:

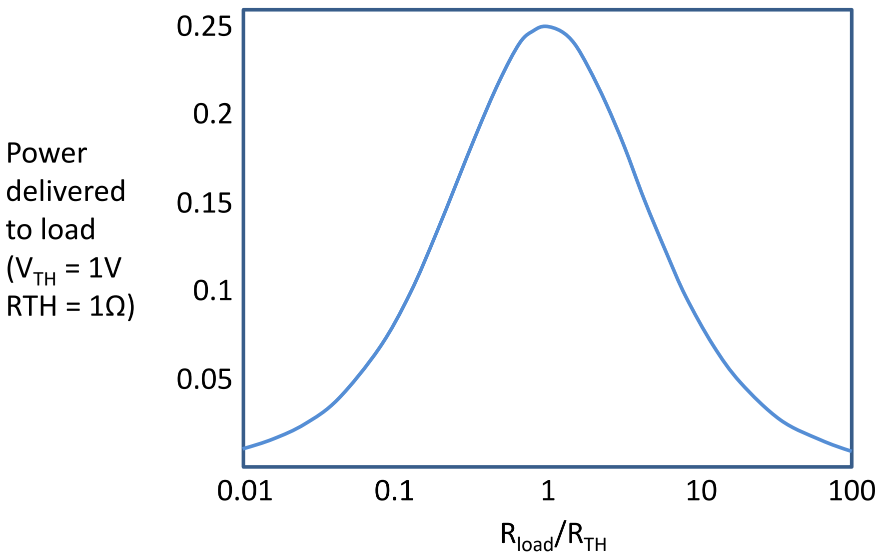

Figure 7.4 Plot of power delivered to load against load resistance

All we have to do is find the maximum value of the graph of P against R_{load}.

Differentiating equation (7.13) with respect to R_{load} gives:

$$\frac{dP}{dR_{load}} = {V_{TH}}^2 \frac{\left(R_{TH} + R_{load}\right)^2 - 2R_{load}\left(R_{TH} + R_{load}\right)}{\left(R_{TH} + R_{load}\right)^4} \tag{7.14}$$

and setting this equal to zero for a turning point reveals that:

$$\frac{dP}{dR_{load}} = V_{TH}{}^2 \frac{\left(R_{TH} + R_{load}\right)^2 - 2R_{load}\left(R_{TH} + R_{load}\right)}{\left(R_{TH} + R_{load}\right)^4} = 0$$

$$\left(R_{TH} + R_{load}\right)^2 = 2R_{load}\left(R_{TH} + R_{load}\right) \quad (7.15)$$

$$R_{TH} + R_{load} = 2R_{load}$$

$$R_{TH} = R_{load}$$

In words: the maximum power delivered to the load (in this case the headphones) occurs when the resistance of the headphones is equal to the output resistance of the amplifier driving the circuit.

7.5.1 A common mistake to avoid

There's a trick question that can be asked at this point. Suppose you are given a pair of headphones with an impedance of 100 ohms. You are asked to design an amplifier to drive these headphones, with the amplifier powered from specified power supply. What should the output impedance of the amplifier be, in order to deliver the maximum possible power to the headphones?

The answer is not 100 ohms. The answer is zero.

What happened to the maximum power theorem? Didn't we just work out that to get the maximum power into a load, the resistance of the load should be equal to the output resistance of the source (in this case the amplifier)?

What's wrong here is that the maximum power theorem only tells you what the load should be, given a known, fixed output impedance from the source. It does not tell you what the source impedance should be, given a known, fixed load.

If you actually have a fixed, known load, then the circuit you're building is effectively yet another potential divider:

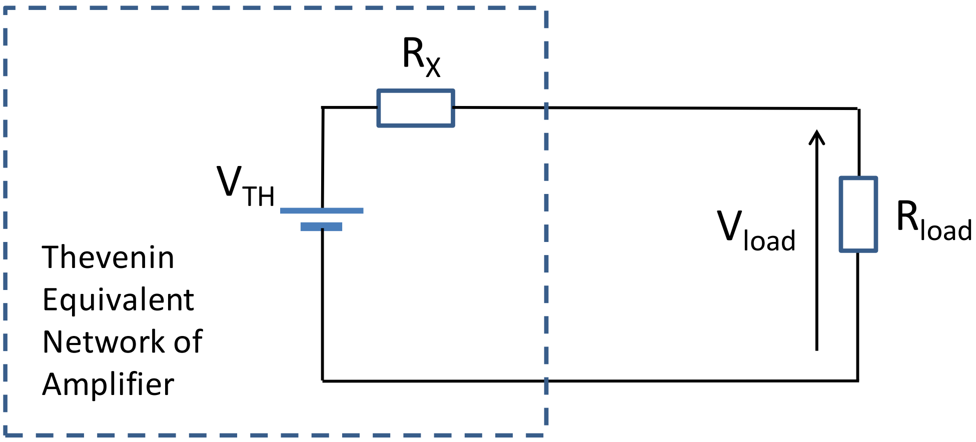

Figure 7.5 Effective circuit of an amplifier driving a given, fixed load

and the power delivered to the output load is:

$$
\begin{aligned}
P = V \times I = V_{TH} \frac{R_{load}}{R_X + R_{load}} \times \frac{V_{TH}}{R_X + R_{load}} \\
= {V_{TH}}^2 \frac{R_{load}}{\left(R_X + R_{load}\right)^2}
\end{aligned}
\qquad (7.16)
$$

and it's not difficult to see that the maximum value the power can have is when the output impedance of the amplifier (here called R_X) is zero[4].

7.6 Summary: the most important things to know

- The output voltage of a circuit can (and usually does) change depending on what is connected to the output.
- The effect can be calculated using the potential divider equation.
- If a source has an output resistance of R, the maximum power it can deliver into a load occurs when the load has a resistance of R.

[4] At least assuming that you can't set it to a negative resistance value, which is usually the case.