1 A Short Introduction to Newtonian Physics

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Prerequisite knowledge required: None

1.1 Introduction

You could argue that fundamentally, electronics (in fact all engineering) is just applied physics. Certainly there are a lot of concepts from physics that are used throughout electronics, and this short note is designed to act as a quick introduction to the most important principles in Newtonian physics[1]. A basic understanding of these principles and concepts such as energy and power are essential to understanding any electronic circuit.

The topics I'll cover in this chapter are energy, work, power, force, pressure and flow. I'll mostly do this in the context of plumbing (water moving around pipes), since there are a lot of parallels between plumbing and electronics[2], but plumbing is probably easier to grasp since you can actually see the water flowing around the pipes and feel the pressure of the water on your hands. I'll finish with a quick description of how all this relates to electronics.

Important point: I'm going to neglect the effect of gravity in this discussion. Please assume that all pipes and other components are lying on a flat horizontal surface.

1.2 Newton's laws of motion

Everything starts here. Isaac Newton's famous three laws of motion that overturned centuries of belief that to move, a body must be feeling some propelling force to keep it moving[3]. These laws are:

1. A body remains at rest or in a state of uniform motion unless acted on by an external force.
2. The rate of change of momentum[4] is equal to the net force applied.
3. When one body exerts a force on another body, there is an equal and opposite force applied by the second body on the first body.

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[1] The "Newtonian" in the title of this note is a reference to Isaac Newton, who did so much to develop the laws of physics in the 17th and 18th centuries. He derived the laws describing the motion of solids in bulk travelling at speeds much less than the speed of light; laws which work as good approximations in most cases even today when we know they are not quite right, and certainly not in the cases of very small particles (for which we need quantum mechanics) or very fast speeds (the domain of relativity).

(We'll come across examples when we'll need some ideas from both quantum mechanics and relativity quite soon, but for now I'll just stick to the basic principles as set down by Isaac Newton.)

[2] It's known as the "hydraulic analogy", see http://en.wikipedia.org/wiki/Hydraulic_analogy

[3] Quite understandable really, since in the experience of the ancient Greeks, things did seem to stop moving just by themselves, e.g. balls rolling along the ground, and arrows shot through the air. So it's not unreasonable to think that there must be some force acting to keep them moving. Newton's brilliant insight was to realise that this is not true, as stated in his first law.

[4] Momentum is the product of mass and velocity. So, for example, a mass of 10 kg moving at 1 m/s and a mass of 100 g moving at 100 m/s have the same momentum: 10 kg m/s.

The second law is the most useful one for us: the rate of change of momentum is equal to the net force applied. Since for most cases of interest the mass of moving bodies doesn't change, we can re-write this in the form "the net force is equal to the mass times the rate of change of velocity", and since the rate of change of velocity is the acceleration (by definition of acceleration), we get the famous formula:

$$F = m \times a \tag{1.1}$$

where *F* is a force (measured in newtons, named in honour of Isaac Newton) exerted on a body of mass *m* kg, producing an acceleration[5] of *a* m/s^2.

The other really important result to know before we get started is that the energy input into a system (sometimes called the work done) is equal to the force times the distance over which the force moves:

$$W = F \times d \tag{1.2}$$

where *W* is the work done (measured in joules), and *d* is the distance the object moves (in metres).

This formula shows that in theory, a force pushing up against a brick wall, no matter how strong the force is, requires no energy and does no work, since the brick wall doesn't move[6].

With these two formulas, we're ready to move into the world of plumbing.

1.3 Flow

I'll start with an easy one: flow. The flow of water in a pipe (measured in litres per second) is just the amount of water that passes any fixed point in a pipe in one second.

There are two different ways to get a large flow through a pipe: either get the water to move very quickly through the pipe, or replace the pipe with a much wider pipe that can carry more water. In both cases, the flow (the amount of water which passes a fixed point in a second) is the cross-sectional area of the pipe times the velocity of the water.

If that's not obvious, consider Figure 1.1: in both cases, the cross-sectional area of the pipes are labelled *A* and the distance that water in the pipe travels in one second[7] is marked *d*. So in a time of one second, a volume $V = A \times d$ of water flows past any fixed point, and this is the flow rate.

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[5] Acceleration is measured in units of meters per second per second, or meters per second^2. A body accelerating with an acceleration of *a* means that its velocity is increasing by *a* m/s every second.

[6] Unless the brick wall falls over, in which case work is being done and energy is required.

[7] The distance travelled in one second is just the velocity, so this length *d* is numerically equal to the velocity of the water. This means that the flow rate can be (and often is) written as $V = Av$ where *v* is the velocity and *A* is the cross-sectional area of the pipe.

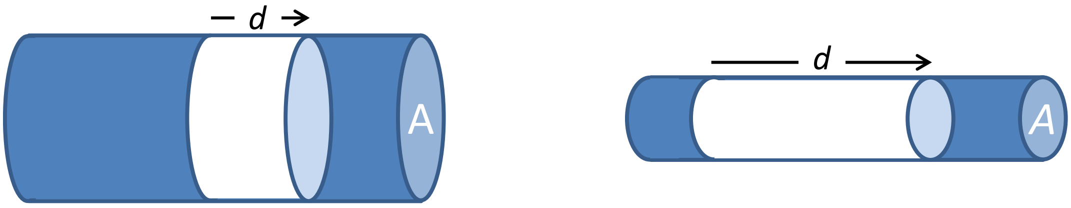

Figure 1.1 Wide and narrow pipes with the same flow rate

So far so good; however what causes water to move along a pipe? Why does it go in one direction but not the other? To answer that one, we need the next concepts: force and pressure.

1.4 Force

A force is either a push or a pull that acts to try to change the motion of an object. In the case of water, you can exert a force by pushing a piston into a cylinder of water, something like this:

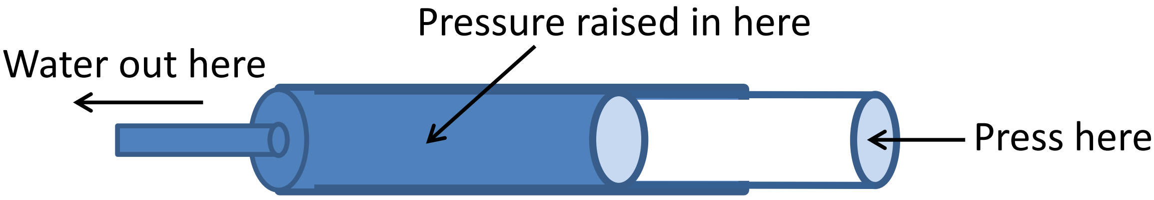

Figure 1.2 Piston raising the pressure in a cylinder

Pushing the piston in will put a force on the water trying to make it leave the piston and flow along the pipe. You can also pull the piston out, and that will result in water moving into the cylinder from the pipe.

Force is measured in newtons (N)[8]. It was Newton that realised that to change the velocity of an object (either to speed it up or slow it down) a force must be acting on the body. In the case of the piston here, the force is causing the water to start moving along the pipe. The greater the force, the faster water emerges from the piston.

1.5 Pressure

Pressure is the ratio of force to area. It's measured in pascals (Pa). If the piston has a cross-sectional area of *A*, and if a force of *N* newtons is exerted on the end of the piston (where it says "Press here" in the figure), then pressure in the cylinder will have increased by *N*/*A* pascals:

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} \tag{1.3}$$

If the water is not able to move (there is no way for it to get out of the pipes, and the pipes are not elastic so the volume inside the network of pipes cannot expand) then the pressure of the fluid throughout the entire system of pipes will be the same.

[8] Outside the realm of hydraulics, a force of one newton is defined as the force required to accelerate a one kilogram mass by one meter per second every second that the force is acting.

However if the pressure is different in different parts of the system, then the water will always try to move from the higher-pressure region to the lower-pressure region. (You can think of pressure as a measure of how much water hates being somewhere. The higher the pressure, the more the water wants to go somewhere else.)

1.5.1 A two-cylinder thought experiment

Suppose you have two cylinders of different cross-sectional areas, with a narrow pipe connecting them (see Figure 1.3). You exert the same force on both of them. What happens?

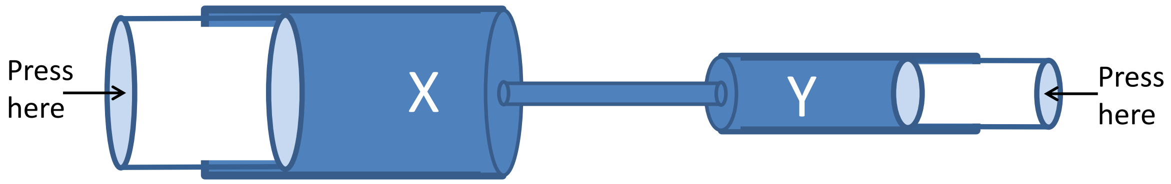

Figure 1.3 A thought experiment with two cylinders

The answer is that the water moves from the cylinder marked 'Y' to the cylinder marked 'X'. Why? Because the area over which the force is exerted in cylinder Y is less than the area over which the same force is exerted in cylinder X. Therefore the pressure in cylinder Y is greater than the pressure in cylinder X, and water will always move from a higher pressure point to a lower pressure point (if it can).

So the cylinder Y would start to empty, and the cylinder X would start to fill up.

At this point, it's worth asking what the pressure is inside the small pipe connecting the cylinders. The answer is that it depends where in the small pipe you are. It's equal to the pressure in cylinder Y at the right-hand end, and equal to the pressure in cylinder X at the left-hand end. In-between, the pressure changes linearly between these values. This is what small pipes do: they restrict the flow of water and maintain pressure gradients along themselves.

1.6 Energy

Energy is a very important concept in physics. Anything that moves or has the potential to move has energy, and transferring energy between different elements in a system is what causes interesting things to happen (sounds to be produced, lights to come on, etc).

There are two distinct types of energy: potential energy and kinetic energy. Both are measured in joules (J). One joule is the energy required to move a piston though one meter while exerting a force of one newton on the piston. (It doesn't have to be a piston, it could be anything.)

Potential energy is the energy that an object has by virtue of its position in space, and it is called *potential energy* because this energy has the potential to be changed into something useful: it has the potential to do work. Kinetic energy is the energy that an object has by virtue of the fact that it's moving.

While you can convert potential energy into kinetic energy and back again, the total energy in a system remains constant: this is known as the principle of conservation of energy[9]. For example, consider a pendulum:

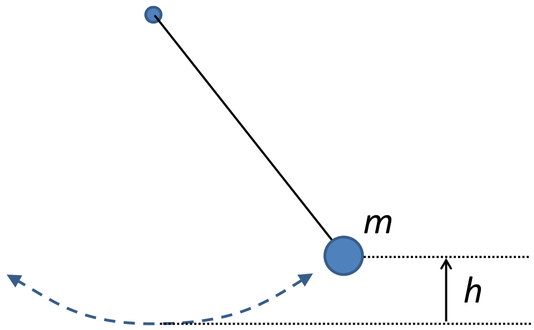

Figure 1.4 A pendulum at the top of its swing

At the top of the swing, the mass on the end of the string has a greater height (and hence greater potential energy, since potential energy increases with height), however it has no kinetic energy since it is not moving. At the bottom of the swing, the mass has less potential energy (since it is lower down), but it now has some kinetic energy (since it's now moving).

By the principle of conservation of energy, you can calculate how fast the pendulum must be moving at the bottom of the swing given how far up it reaches at the top of the arc. The first thing you need to know about is the force on an object in the Earth's gravitational field. It is an observable fact that an object in the lab, if left to fall in a vacuum (so there is no air resistance), accelerates towards the Earth with an acceleration of g m/s^2, where g is a constant acceleration known as the *acceleration due to gravity*.

Applying Newton's second law suggests that the force on this object, pulling it towards the Earth is mg (the mass times the acceleration). If you want to lift an object up, then you have to act against that force pushing down. That takes work: and the work required is the force times the distance moved. So if the object is lifted up through a height h, the work required to be done must be:

$$W = mg \times h \tag{1.4}$$

Where does this energy go? It goes to increase the potential energy of the object (the energy "stored" in the object). Therefore the potential energy must increase by an amount:

$$E_P = mgh \tag{1.5}$$

In the case of the pendulum, E_P is the potential energy, m is the mass at the end of the string, g is the acceleration due to gravity (about 9.8 ms^{-2}) and h is the height of the object reached above the bottom of the arc.

[9] While widely believed to be exactly true for a long time, we now know it's not quite true once the theory of relativity is considered. In modern physics, new energy can be formed by converting mass into energy (the famous equation $E = mc^2$ quantifies this relationship).

Now if the mass is released, it will fall through the height h until it gets to the bottom of the arc. At this point, it has lost energy equal to *mgh*. This energy has been transferred to kinetic energy: the energy the object has by virtue of the fact that it is moving.

To work out how fast the object is going, a formula for kinetic energy is required. One of the equations of motion (under constant acceleration) is:

$$v^2 = u^2 + 2as \tag{1.6}$$

where *v* is the final velocity, *u* is the initial velocity, *a* is the acceleration and *s* is the distance moved. Suppose an object was dropped from a height *h*: the initial velocity would be zero, and the acceleration would be *g*, so the final velocity would be given by:

$$v^2 = 2gh \tag{1.7}$$

Multiply both sides by *m*/2, and we get:

$$\frac{1}{2}mv^2 = mgh \tag{1.8}$$

The right-hand side of equation (1.8) is the potential energy lost, and by the principle of conservation of energy this must be equal to the kinetic energy gained. So the kinetic energy of a moving object must be given by:

$$E_K = \frac{1}{2}mv^2 \tag{1.9}$$

Applying these ideas to the pendulum suggests that the energy lost due to the mass falling from the top of the swing to the bottom of the swing must equal the kinetic energy at the bottom of the swing, so:

$$\begin{aligned} mgh &= \frac{1}{2}mv^2 \\ v &= \sqrt{2gh} \end{aligned} \tag{1.10}$$

where *h* is the height difference between the top and the bottom of the swing.

In the plumbing analogy, the water flowing along the pipes has kinetic energy, and water under pressure has potential energy.

1.7 Work

Work, like energy, is measured in joules. However, while the term *energy* is conventionally used for the energy stored within a system or object, *work* is more usually used to apply to energy moving from one system to another: energy flowing into a circuit for example, or energy leaving a circuit to do something useful (move an object, make a sound, produce some light, etc).

Think back to Figure 1.3, and consider the work done by the person pressing on the smaller cylinder. If the force being applied is *F*, and the distance the piston moves is *d*, then the work done is given by:

$$\text{Work Done} = \text{Energy Put Into System} = F \times d \tag{1.11}$$

since the energy is the force times the distance. This can be usefully re-cast in terms of the pressure and the cross-sectional area, given that:

$$\text{Pressure} = \frac{\text{Force}}{\text{Cross-sectional Area}} \tag{1.12}$$

and:

$$\text{Distance} = \frac{\text{Volume of Water}}{\text{Cross-sectional Area}} \tag{1.13}$$

which gives:

$$\text{Work Done} = F \times d = (p \times A) \times \left(\frac{V}{A}\right) = p \times V \tag{1.14}$$

where p is the pressure in the cylinder, A is the cross-sectional area of the piston, and d is the distance travelled. So the work done in moving a volume of water along a pipe is just the volume of water moved multiplied by the pressure the water is under.

1.7.1 Work and energy in the thought experiment

At this point, an example might be interesting. Think back to the thought experiment with the two pistons, both of which were being pushed in with the same force (see Figure 1.3). What is happening in terms of work and energy?

As already noted, with equal forces applied, the smaller piston will exert a higher pressure, and the larger cylinder a lower pressure, so the water will be moving from the smaller cylinder to the larger one.

The smaller piston is then moving in the direction of the force being applied, and this means that energy is required to do this (in other words work is being done to move this cylinder). As noted above, the work done is equal to the pressure being applied times the volume of water moved.

The larger piston is moving against the force: the force is trying to push the piston in, but the piston is actually being moved out. So energy can be extracted from the system here: you could attach a generator to produce power from this movement being made against a force. The energy extracted is again the pressure times the volume, however in this cylinder the pressure is lower (which is why the water is moving in the first place).

However, the volume must be the same: for every molecule of water leaving the smaller cylinder, one will arrive in the larger cylinder. So more work is being done by the smaller cylinder than energy could be extracted by the large cylinder. But energy is conserved, so where did the rest of the energy go? The answer is that it would heat up the water in the small pipe as the water moves through it.

1.7.2 Work and energy in a closed loop

As we're just seen, in terms of plumbing, you put energy into the system by raising the pressure of the water. A pump will take energy in from an outside source (usually electric power), take water in at low pressure, and use the energy to increase the potential energy in the water by raising the pressure of the water, outputting water at a higher pressure.

The work done by the pump is the energy in the water leaving minus the energy extracted from the water arriving:

$$\text{Work Done} = p_{out} \times V_{out} - p_{in} \times V_{in} \tag{1.15}$$

where p_{in} and p_{out} are the pressures at the input and output of the pump, V_{in} is the volume of water flowing into the pump, and V_{out} is the volume of water flowing out.

Since water is (to a good approximation) incompressible, the volume of water flowing in must be the same as the volume flowing out, so we could just call this V, and hence write:

$$\text{Work Done} = \left(p_{out} - p_{in}\right) \times V \tag{1.16}$$

or in words: the work done by the pump in any given time is the difference in pressure between the input and output multiplied by the volume of water that flows through.

At this point you might be a little confused about something: suppose there is a pump and a pipe arranged as shown below. The pump uses energy to raise the pressure of the water before outputting it. What happens if you take the water from the high-pressure end of the pump, and connect it directly round to the low-pressure end of the pump?

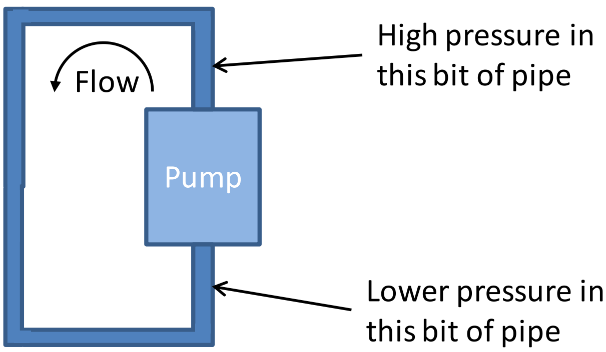

Figure 1.5 A pump with its output connected directly to its input

You might think that the pump would increase the pressure in the water at its output, and this higher pressure would move round to the input, so the pump would then increase the pressure again, and so on, as the pressure in the closed system continually increases without limit.

In theory, yes, this is what would happen. In practice two things would happen to prevent this. Firstly, all pumps have a maximum pressure that they can give at their output, and when they reach that pressure the mechanism of the pump itself stops working. At this point the pump is not increasing the pressure of the water, so it is doing no work, and therefore requires no energy.

Secondly, and more significantly, not all of the pressure at the output of the pump would get back round to its input. As the water is moving through the pipe, from the output to the input, it is constantly losing some pressure.

If you think about it, this has to happen. If there is no difference in pressure between the water at the output of the pump and the water at the input to the pump, then the water in the pipe would not move: there would be no net force on the water. For all real pipes, if there is water flowing through them, then the pressure at the output end of the pipe must be lower than the pressure at the input end of the pipe.

This is important enough to repeat: if water is moving along a pipe, then the pressure is not constant along the pipe. In fact, the difference in pressure between the two ends of the pipe is often linearly proportional to the rate of flow along the pipe.

We can express this as an equation:

$$\left(p_{out} - p_{in}\right) \propto Flow \tag{1.17}$$

The difference in pressure between the water at the two ends of the pipe is proportional to the rate of flow through the pipe. The constant of proportionality is related to the width and the length of the pipe (smaller and longer pipes have larger constants, shorter and wider pipes have smaller constants).

1.7.3 Conservation of energy

I noted before that there is a rule that energy is conserved: it can neither be created nor destroyed, it just changes form. However, in the example above, there is a pump constantly doing work to raise the pressure of the water flowing around the loop of pipe. This pump is putting energy into the plumbing system. Where does that energy go?

The answer is (mostly[10]) heat: the water will heat up as it passes through the pipes from a higher-pressure point to a lower-pressure point in the plumbing circuit.

1.8 Power

Power is the rate of using (or producing) energy. It's measured in Watts[11] (W) where one watt is the rate of one joule of energy (or work) per second.

This can be taken one step further: the work done per second is the power being put into the system (power is just energy per second), and the volume being moved per second is the rate of flow, so:

$$\begin{gathered}\text{Work Done per second} = \left(p_{out} - p_{in}\right) \times \text{Volume per second} \\ \text{Power} = \left(p_{out} - p_{in}\right) \times \text{Flow}\end{gathered} \tag{1.18}$$

[10] I say "mostly" since there will be a little lost to sound: you can usually hear a pump working, and water flowing through pipes. However the vast majority will be lost to heat.

[11] Named after James Watt, who being Scottish had no chance of getting his name on the Eiffel tower, although he does have a statue in St. Paul's Cathedral, and a memorial stone in a supermarket car-park.

In this plumbing system, the power being input into the system by the pump is the difference in pressure (in pascals) between the input and output of the pump, times the flow (in litres per second).

1.9 Back to electronics

Why spend so much time talking about plumbing? Because if you substitute “battery” for “pump”, “charge” for “water”, “voltage” for “pressure”, “current” for “flow”, “wire” for “thick pipe” and “resistor” for “narrow pipe”, then electronics works much the same way. The analogy isn’t perfect, but it’s a good place to start: think about voltage as a sort of electric pressure, and charge as a sort of incompressible fluid which runs round wires.

The word “round” in that last sentence is particularly important. If we assume that water is incompressible[12] (i.e. its volume doesn’t change when the pressure changes) and electric charge behaves in the same way, then it becomes clear that electric current, just like water in a network of pipes, must flow around loops (otherwise known as circuits). It can’t just flow from one place to somewhere else; there wouldn’t be any more room for the new water/charge when it arrives. Water and charge in these networks have to flow around circuits.

1.9.1 Resistance

Resistance is what limits the current. Suppose you have a pump (which in the analogy represents a battery) which maintains the pressure (the voltage) at one side of the pump at three units above the other end.

If there wasn’t a circuit, but just a pump connected to two bits of pipe with their ends sealed, then all you’d have is this:

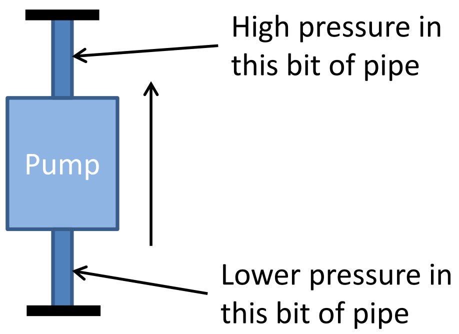

Figure 1.6 Pump with pipe ends sealed (no current)

No current would flow; there is just a pipe with a pump that keeps the pressure at one end higher than the pressure at the other end. Once it’s established that pressure difference nothing moves, which means no more energy is required.

However, connect the high-pressure end to the low-pressure end with a narrow pipe, and the water will begin to flow:

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[12] This isn’t quite true of water, and it’s not quite true of electric charge either. However it’s a very good first approximation for small circuits at low frequencies (the so-called *lumped element* model).

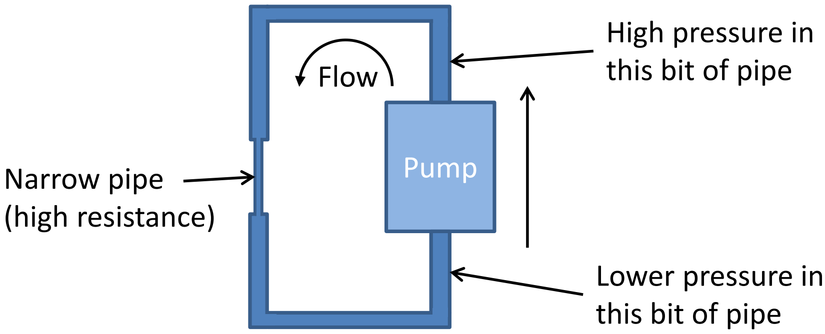

Figure 1.7 Pump with the resultant flow around a closed loop (circuit)

The speed at which the fluid will move around the system is determined by how narrow the pipe is. A very narrow pipe will limit the rate at which current can flow; this is called having a high resistance. On the other hand, a wide pipe allows much more water to flow, and this is analogous to a small resistance. In electronic circuits, resistance is measured in Ohms.

This is a very important idea: the greater the resistance, the less the current that flows. This can be expressed in perhaps the most useful equation of electronics, one that relates the resistance to the current flowing through the resistor and the voltage across the resistor:

$$\begin{gathered}\text{Voltage Difference} = \text{Current} \times \text{Resistance} \\ \Delta V = IR\end{gathered} \tag{1.19}$$

The voltage across a resistor (in the analogy the difference in pressure between one end of the narrow pipe and the other) is the product of the resistance (a measure of the width and length of the pipe) and the current (the flow of water). I've used ΔV here to represent the difference in voltage between the two ends of the resistor, I to represent the current flowing through the resistor and R to represent resistance; again these are the conventional choices. This is known as Ohm's Law, and was one of the first important results to be derived about circuit theory.

1.9.2 Energy and power in electric circuits

We saw before that the work done in moving a volume of water V from a point in the circuit with pressure p_{in} to a point with pressure p_{out} was given by:

$$\text{Work Done} = \left(p_{out} - p_{in}\right) \times V \tag{1.20}$$

and again there is an analogous formula for electric circuits: the work done is given by

$$\text{Work Done} = \left(V_{out} - V_{in}\right) \times Q \tag{1.21}$$

where V_{in} and V_{out} are the voltages[13] at the input and output of a component, and Q is the charge flowing through the component.

[13] Sorry about the confusing use of symbols here. The letter 'V' is commonly used for both volume and voltage, as there isn't usually a need to refer to both in the same equation. Here however, you have to be careful.

Similarly, the current is the rate of movement of charge (the amount of charge that moves per second), and the power provided to a circuit is the rate of doing work, so we can write:

$$\begin{aligned} \text{Work Done per second} &= \left(V_{out} - V_{in}\right) \times Q \text{ per second} \\ \text{Power Required} &= \left(V_{out} - V_{in}\right) \times I \end{aligned} \tag{1.22}$$

and we have another useful formula: the power taken from a battery is the difference in voltages between terminals (the voltage of the battery) multiplied by the current through the battery.

It's important to note that this formula gives the power required from the battery when the current is flowing from the lower voltage to the higher voltage. That's not the direction it wants to go, so you have to do work to make it happen[14].

Across a resistor, on the other hand, the current always flows from the higher voltage end to the lower voltage end. This doesn't require any power to be provided by the resistor; in fact, it releases energy from the circuit as the charge drops in voltage.

The power extracted from the circuit in the resistor (which mostly turns into heat) is given by a very similar formula:

$$\text{Power Released} = \left(V_{in} - V_{out}\right) \times I \tag{1.23}$$

where the input voltage is higher than the output voltage.

You can combine these in this form:

$$\text{Power Supplied to Circuit} = \left(V_{out} - V_{in}\right) \times I \tag{1.24}$$

where V_{out} is the potential where the current emerges, and V_{in} is the potential where the current enters the component. If the power supplied to the circuit is negative, that means that power is leaving the circuit (usually in the form of heat).

Combining this equation with Ohm's Law in equation (1.19) allows two other useful equations to be derived:

$$P = \frac{\Delta V^2}{R} \qquad\qquad P = I^2 R \tag{1.25}$$

where P is the power extracted from the circuit heating up the resistor, ΔV is again the voltage across the resistor ($\Delta V = V_{in} - V_{out}$), R is the resistance of the resistor (the ratio of the voltage across the resistor to the current flowing through it) and I is the current flowing through the resistor.

[14] It is possible to construct circuits in which current flows "backwards" through the battery entering at the high-voltage end and leaving from the low-voltage end. In this case the battery is extracting power from the circuit. This is how rechargeable batteries get charged: they store this energy being provided by the rest of the circuit.

1.10 Summary: the most important things to know

- Everything remains at rest or moving at a constant speed in a straight line unless acted upon by a force.
    - Force is measured in newtons.
    - One newton is the force required to make a one kilogram mass accelerate at one meter per second squared.
- Energy is the capacity to move things around against a force trying to make them move in the other direction (e.g. lift something up, against the force of gravity trying to pull them down). Work is the energy lost when something is actually moved.
    - Energy and work are measured in joules.
    - One joule is the energy required to move something one meter against a force of one newton trying to push it the other way.
    - Alternatively, one joule is the work done when something is moved by one meter against of force of one newton.
- Pressure is the ratio of force to the area over which is acting.
- Power is the rate of movement of energy, measured in watts.
    - One watt is one joule per second.
    - Moving something by one meter per second against a force of one newton requires one watt of power.